Proposal:

Supplementary Release on Backward Equations $p(h,s)$ for Region 3, Equations as a Functions of h and s for the Region Boundaries, and an Equation $T_{sat}(h,s)$ for Wet Steam of the IAPWS-IF97

K. Knobloch, I. Stöcker, H.-J. Kretzschmar
University of Applied Sciences of Zittau and Görlitz, Germany
Department of Technical Thermodynamics

A. Dittmann
Technical University of Dresden, Germany
Department of Thermodynamics

R. Mareš
University of West Bohemia, Plzen

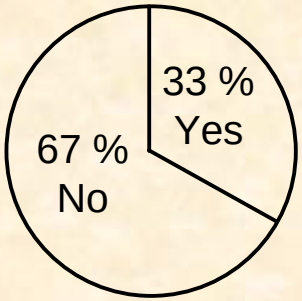
W. Wagner
Ruhr-University, Bochum

IAPWS Meeting, Vejle, 2003

Why Equations as Function of *h* and *s* in Region 3 ?

Kiyoshi Miyagawa: **"Informal survey to ensure the need of industry for backward equations for region 3 of IAPWS-IF97";** IAPWS-Annual Meeting 2002

Do you think IAPWS should provide functions *e.g.* $T(h,s)$, $v(h,s)$, and $p(h,s)$ in region 3 ?



Calculation of the functions $T(h,s)$, $v(h,s)$, and $p(h,s)$ from IAPWS-IF97

$$\boxed{f_3^{97}(T,v)} \longrightarrow \boxed{\begin{matrix} h_3^{97}(T,v) \\ s_3^{97}(T,v) \end{matrix}} \longrightarrow \boxed{p_3^{97}(T,v)}$$

time consuming two-dimensional iteration

Explicit equations for the most important backward functions would be available.

Numerical Consistency Requirements

$$\left|\Delta T\right| = \left| T_3\left[h_3^{97}(T,v), s_3^{97}(T,v)\right] - T \right| \le \left|\Delta T\right|_{tol}$$

$$\left|\Delta T\right|_{tol} = 25\ \text{mK}$$

IAPWS-IF97-S03

$f_3^{97}(T,v)$

$h_3^{97}(T,v)$

$s_3^{97}(T,v)$

$T_3(h,s)$

$v_3(h,s)$

$$\left|\frac{\Delta v}{v}\right| = \left| \frac{v_3\left[h_3^{97}(T,v), s_3^{97}(T,v)\right] - v}{v} \right| \le \left|\frac{\Delta v}{v}\right|_{tol}$$

$$\left|\frac{\Delta v}{v}\right|_{tol} = 0.01\%$$

IAPWS-IF97-S03

First Results

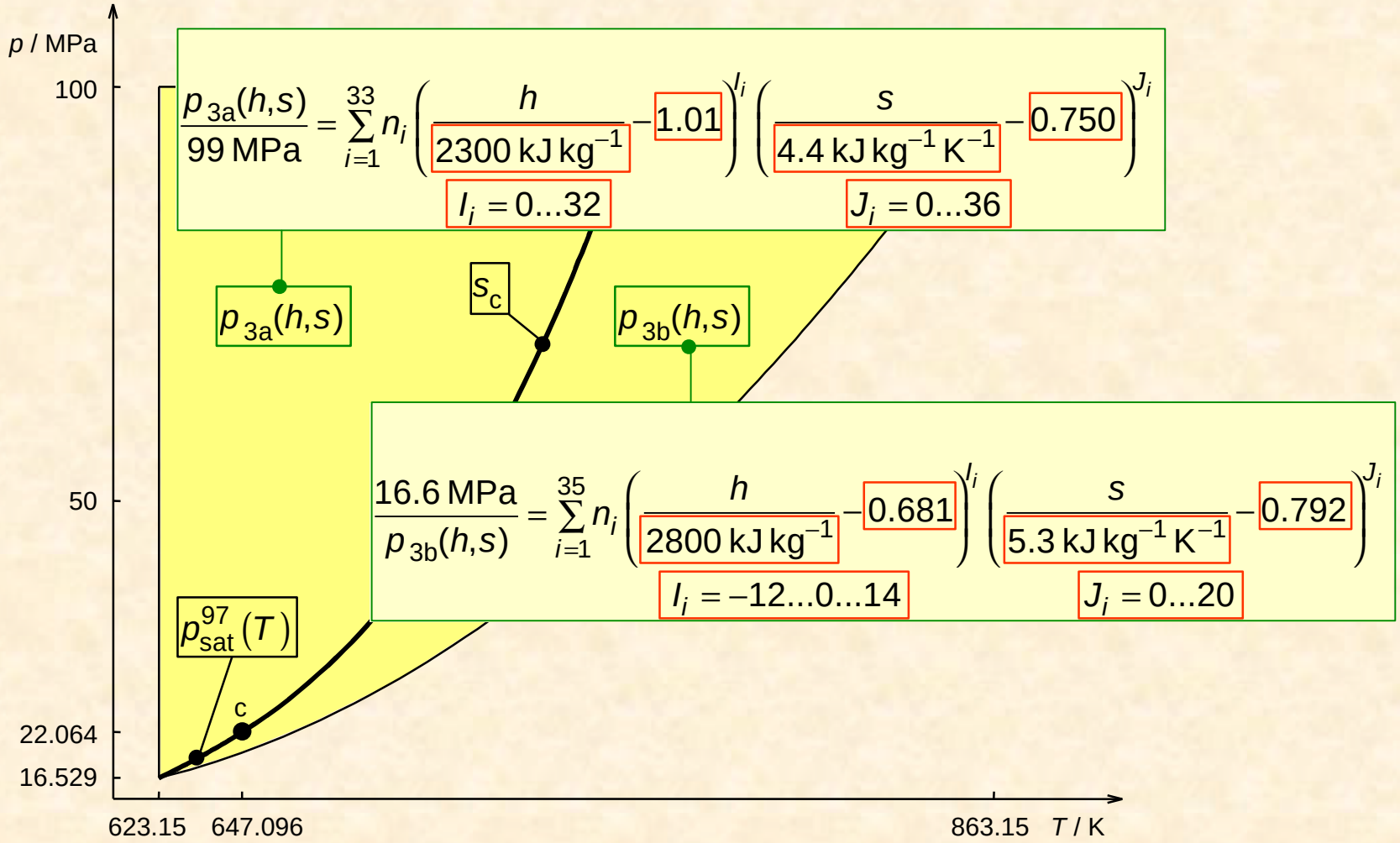
Function	$T_3(h,s)$	$v_3(h,s)$
Achieved accuracy	38 mK	0.09 %

Alternative: Equations $p_3(h,s)$

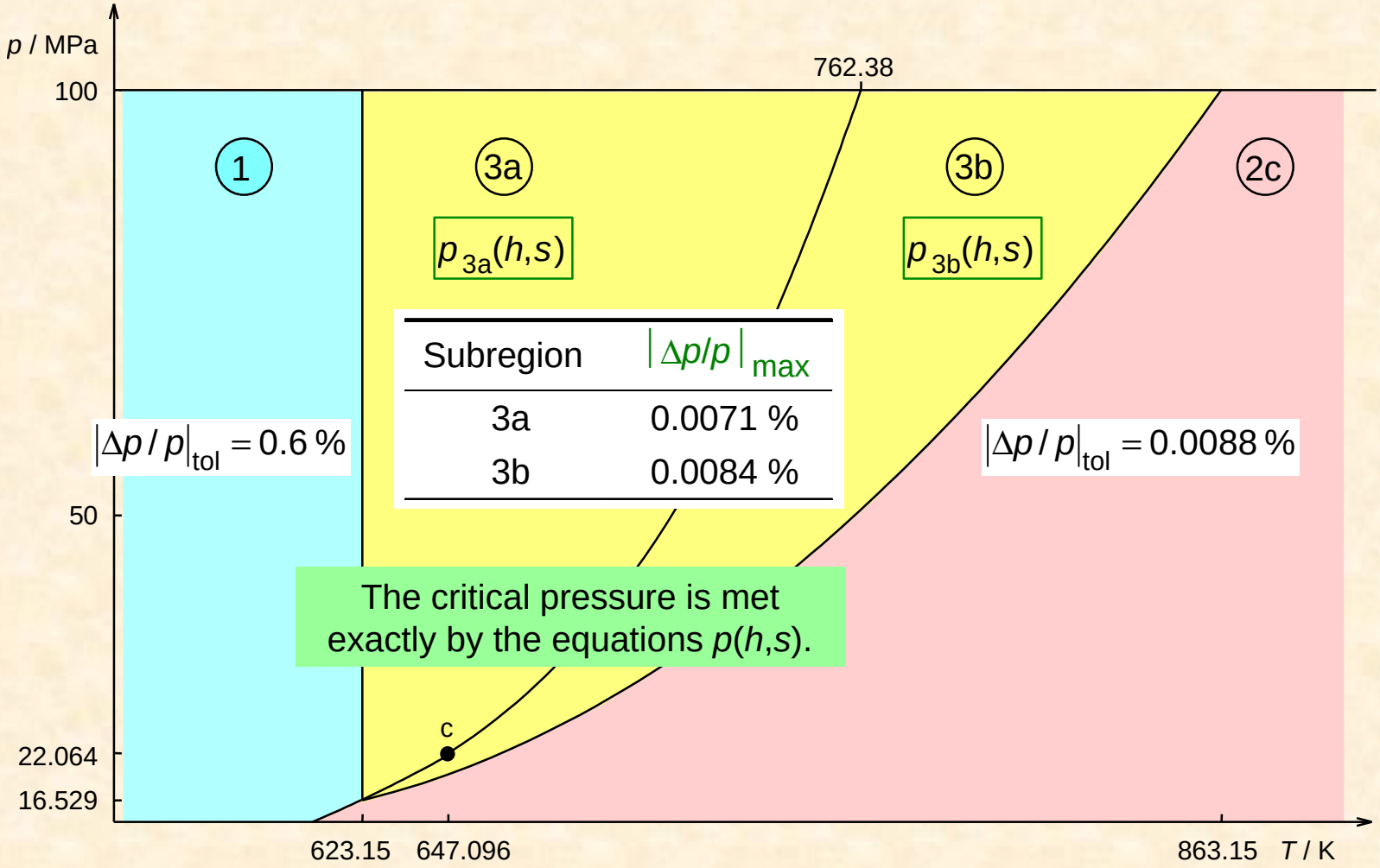
$T_3(h,s)$ with $T_3^{03}(p,h)$ or $T_3^{03}(p,s)$

$v_3(h,s)$ with $v_3^{03}(p,h)$ or $v_3^{03}(p,s)$

Structure of the Equation Set



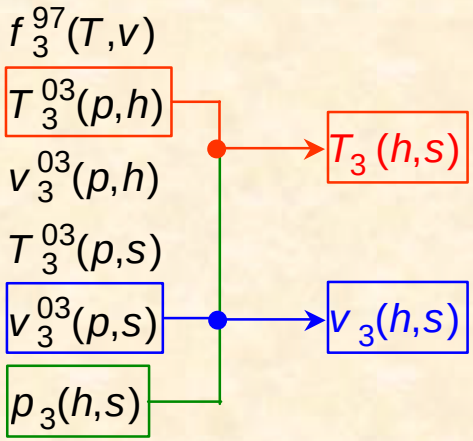
Results for Numerical Consistency with IAPWS-IF97 Basic Equation



Subregion	$\lvert \Delta p/p \rvert_{max}$
3a	0.0071 %
3b	0.0084 %

The critical pressure is met exactly by the equations $p(h,s)$.

Calculation of the Backward Function $T_3(h,s)$ and $v_3(h,s)$



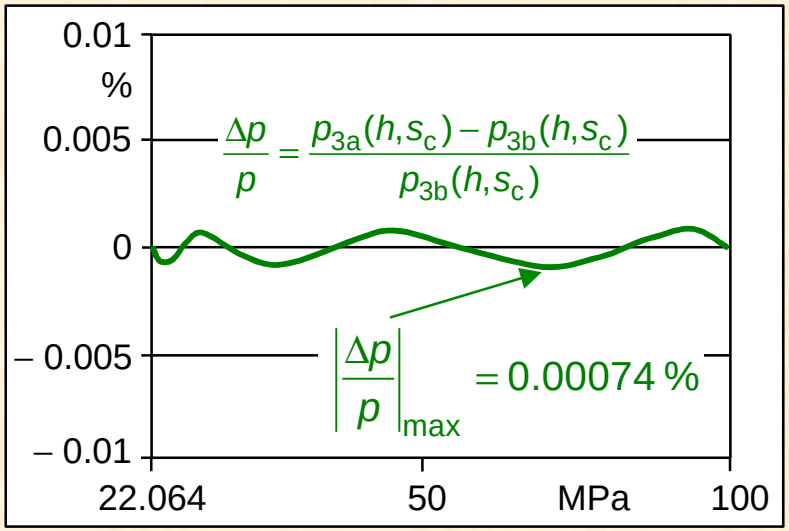
Numerical Consistency with the Basic Equation of IAPWS-IF97

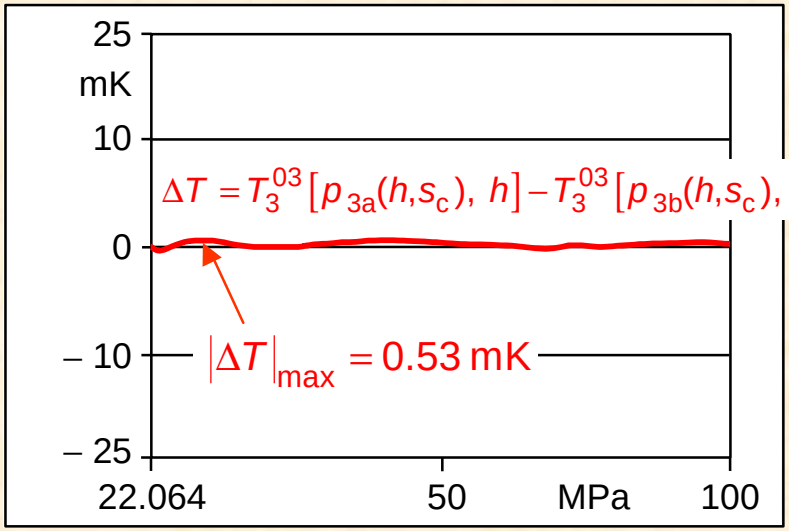
Subregion	$\lvert\Delta T\rvert_{tol}$	$\lvert\Delta T\rvert_{max}$
3a	25 mK	23.7 mK
3b	25 mK	22.4 mK

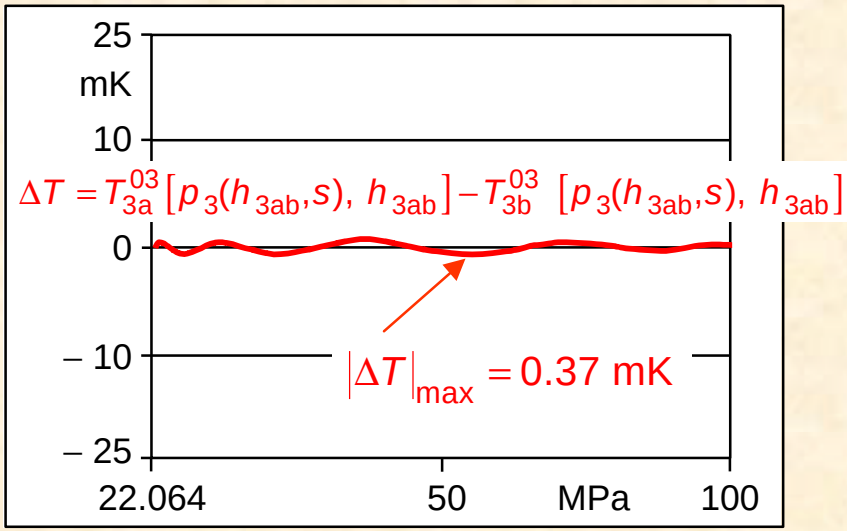
Subregion	$\lvert\Delta v/v\rvert_{tol}$	$\lvert\Delta v/v\rvert_{max}$
3a	0.01 %	0.0097 %
3b	0.01 %	0.0095 %

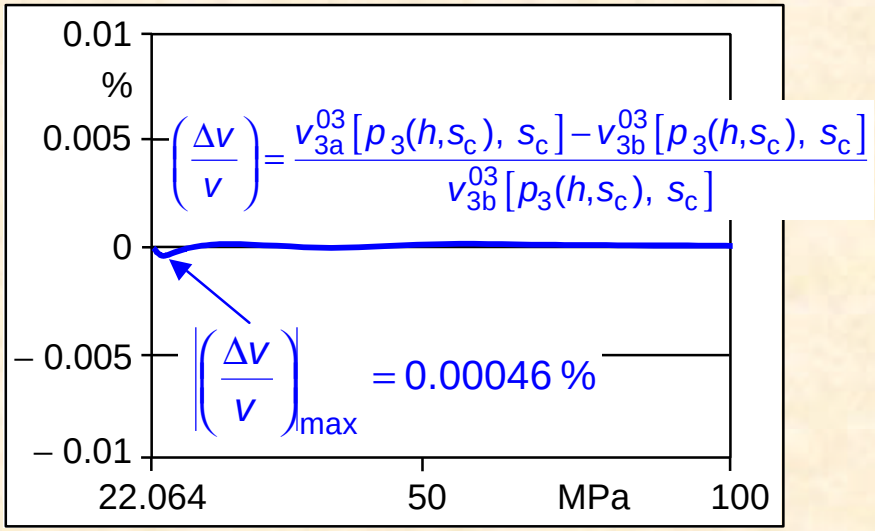
The accuracy of pressure calculated by $p_3(h,s)$ is sufficient for calculating $T_3(h,s)$ and $v_3(h,s)$.

Consistency at Boundary Between Subregions 3a and 3b









Computing Time in Relation to IAPWS-IF97

Calculation of the functions $T(h,s)$, $v(h,s)$, and $p(h,s)$:

by iteration from IAPWS-IF97		by new equations	
Subregion	Computing Time	Computing Time	*CTR*
3a	8.2 μs/call	0.74 μs/call	11
3b	7.9 μs/call	0.73 μs/call	11

$$CTR = \frac{\text{Computing time using IAPWS-IF97 equations only}}{\text{Computing time using the backward equations}}$$

Conclusion - Backward Equations *p*(*h*,*s*) for Region 3

Backward equations *p*(*h*,*s*) for region 3 of IAPWS-IF97 have been presented.

Functions *T* & *v* (*h*,*s*) can be calculated using the *T*(*p*,*h*) and *v*(*p*,*s*) equations of IAPWS-IF97-S03.

⇩

Numerical consistency is sufficient for process modeling.

⇩

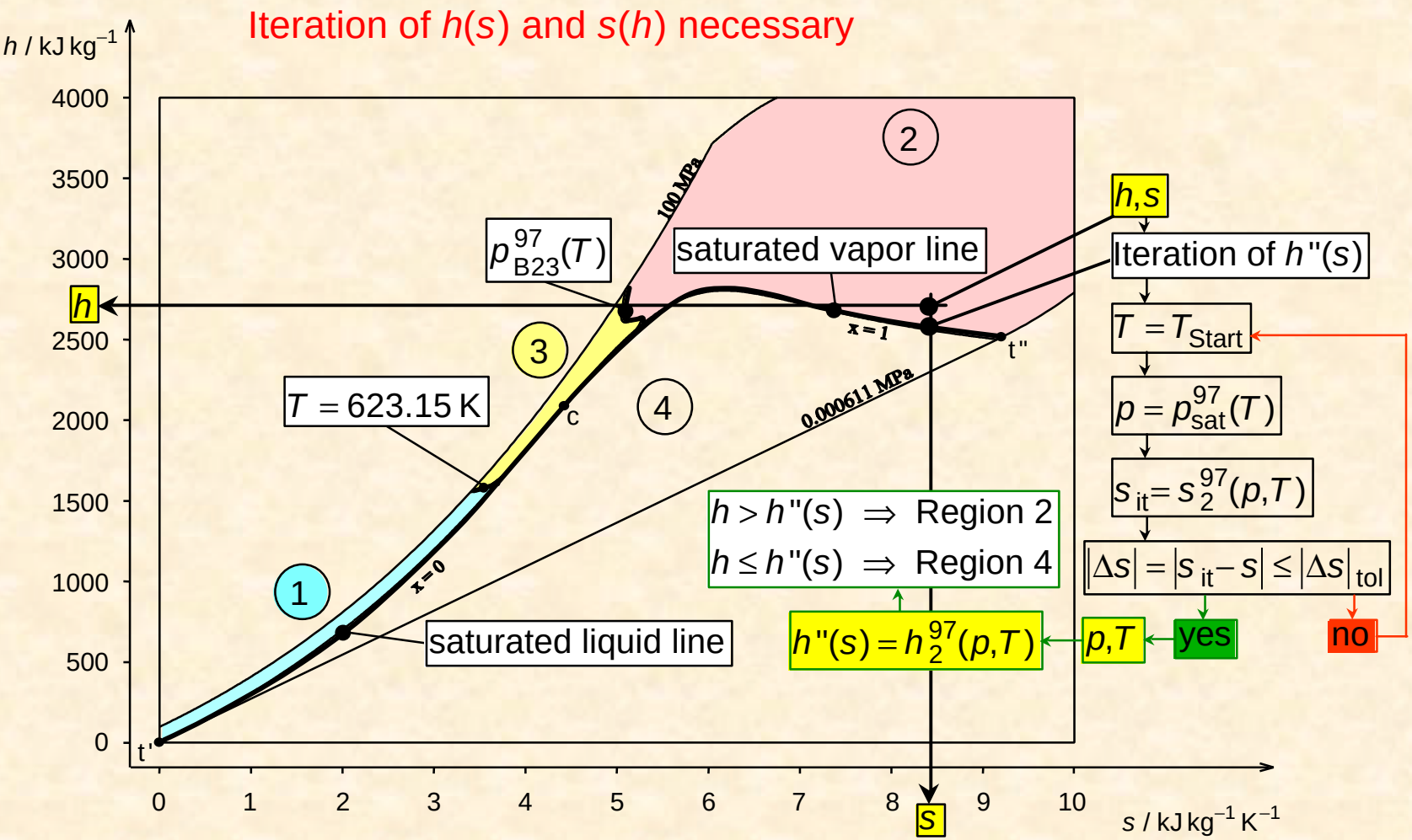
2-dimensional iterations can be avoided.

⇩

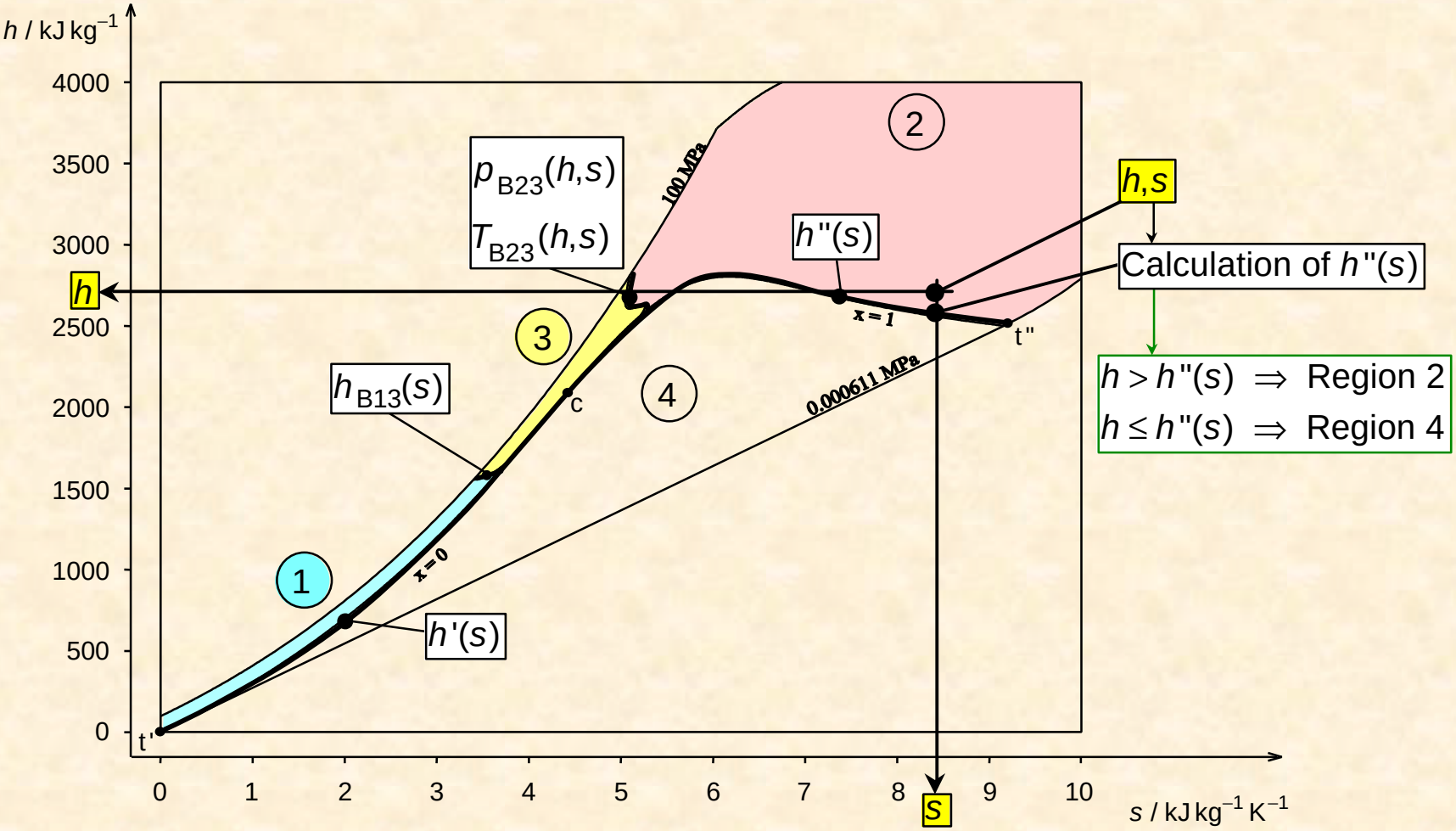
The calculation of *T*, *v* and *p*(*h*,*s*) is 11 times faster than IAPWS-IF97.

Explicit equations for the most important backward functions are available.

Determination of Region Boundaries from Given *h* and *s*



Equations for Region Boundaries from Given *h* and *s*



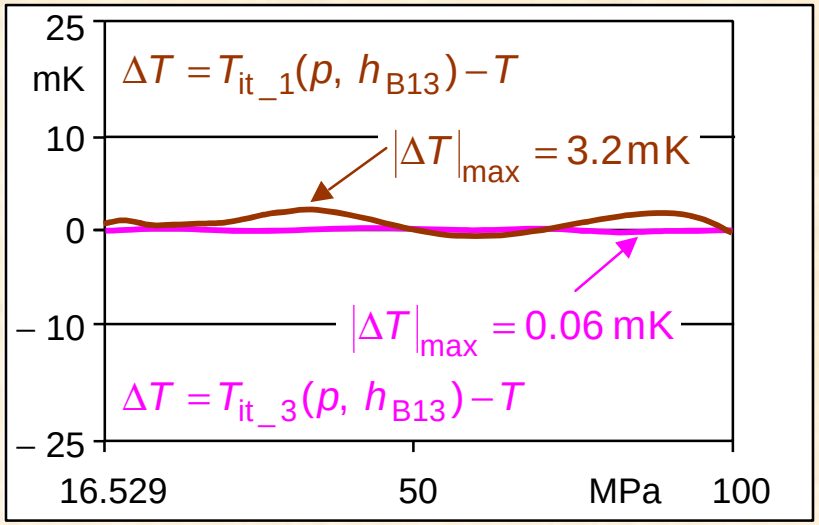
Equation $h_{B13}(s)$ for Boundary between Regions 1 and 3

$$\frac{h_{B13}(s)}{1700\,\mathrm{kJ\,kg^{-1}}} = \sum_{i=1}^{6} n_i \left(\frac{s}{3.8\,\mathrm{kJ\,kg^{-1}\,K^{-1}}} - 0.884\right)^{I_i} \left(\frac{s}{3.8\,\mathrm{kJ\,kg^{-1}\,K^{-1}}} - 0.864\right)^{J_i}$$

$$I_i = 0 \ldots 6 \qquad J_i = -12 \ldots 0 \ldots 4$$

Numerical Consistency with the Basic Equations of IAPWS-IF97

Region	$\lvert \Delta h \rvert_{max}$
1	0.018 kJ kg^{-1}
3	0.00028 kJ kg^{-1}



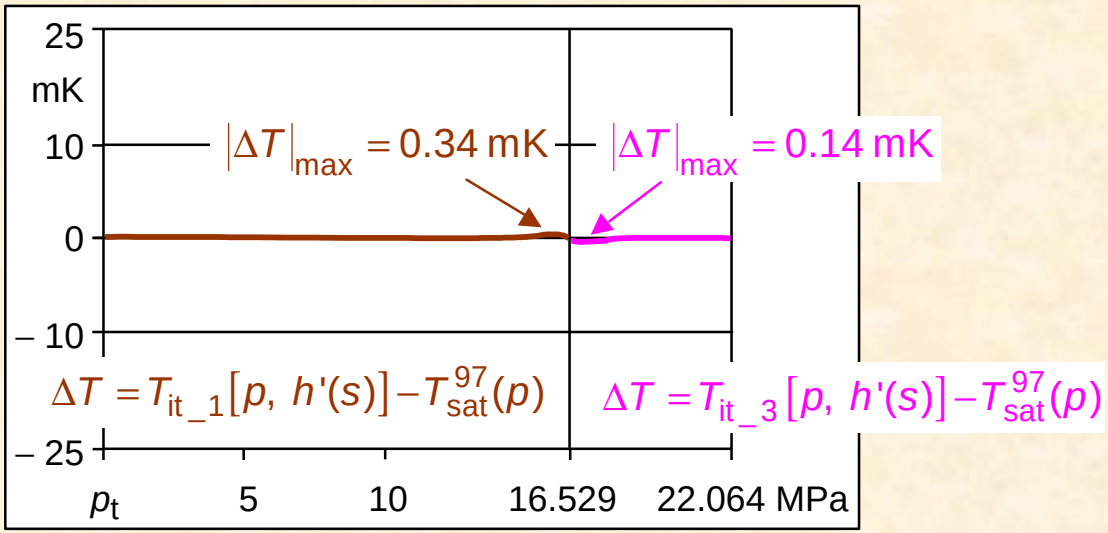
Equation $h'(s)$ for the Saturated Liquid Line

$$\frac{h'(s)}{1700\ \mathrm{kJ\,kg^{-1}}} = \sum_{i=1}^{19} n_i \left(\frac{s}{3.8\ \mathrm{kJ\,kg^{-1}\,K^{-1}}} - 1.09 \right)^{I_i} \left(\frac{s}{3.8\ \mathrm{kJ\,kg^{-1}\,K^{-1}}} + 0.366 \times 10^{-4} \right)^{J_i}$$

$$I_i = 0 \ldots 32 \qquad J_i = 0 \ldots 36$$

Numerical Consistency with the Basic Equations of IAPWS-IF97

Region	$\lvert \Delta h \rvert_{max}$
1	0.0034 kJ kg^{-1}
3	0.0045 kJ kg^{-1}



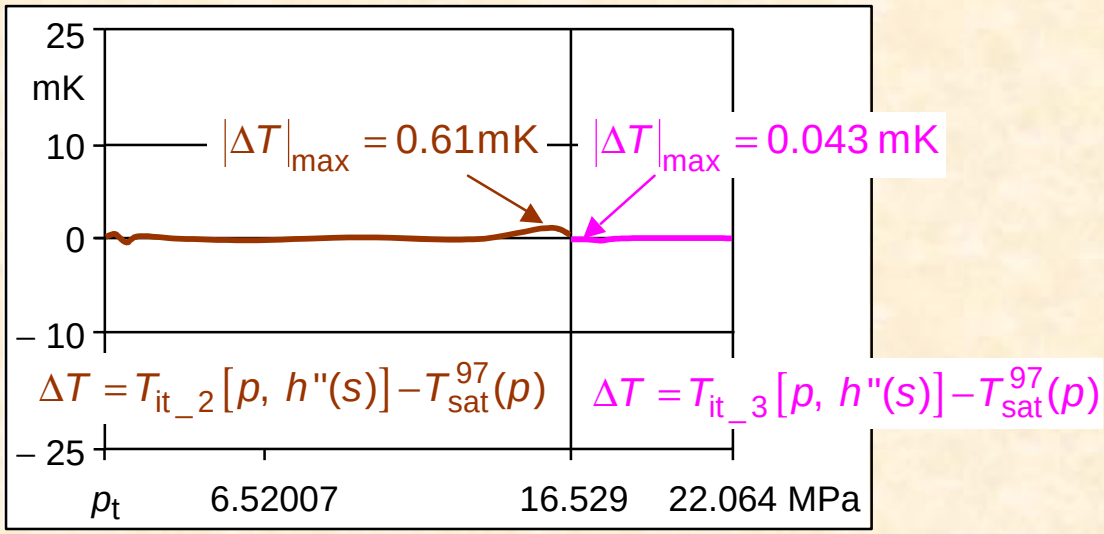
Equation $h''(s)$ for the Saturated Vapor Line

$$\frac{h''(s)}{2800\,\mathrm{kJ\,kg^{-1}}} = \exp\left[\sum_{i=1}^{30} n_i \left(\frac{s}{5.21\,\mathrm{kJ\,kg^{-1}\,K^{-1}}} - 0.513\right)^{I_i} \left(\frac{s}{9.2\,\mathrm{kJ\,kg^{-1}\,K^{-1}}} - 0.524\right)^{J_i}\right]$$

$$I_i = 1 \ldots 36 \qquad J_i = 1 \ldots 32$$

Numerical Consistency with the Basic Equations of IAPWS-IF97

Region	$\lvert \Delta h \rvert_{max}$
2	0.0095 kJ kg^{-1}
3	0.0046 kJ kg^{-1}



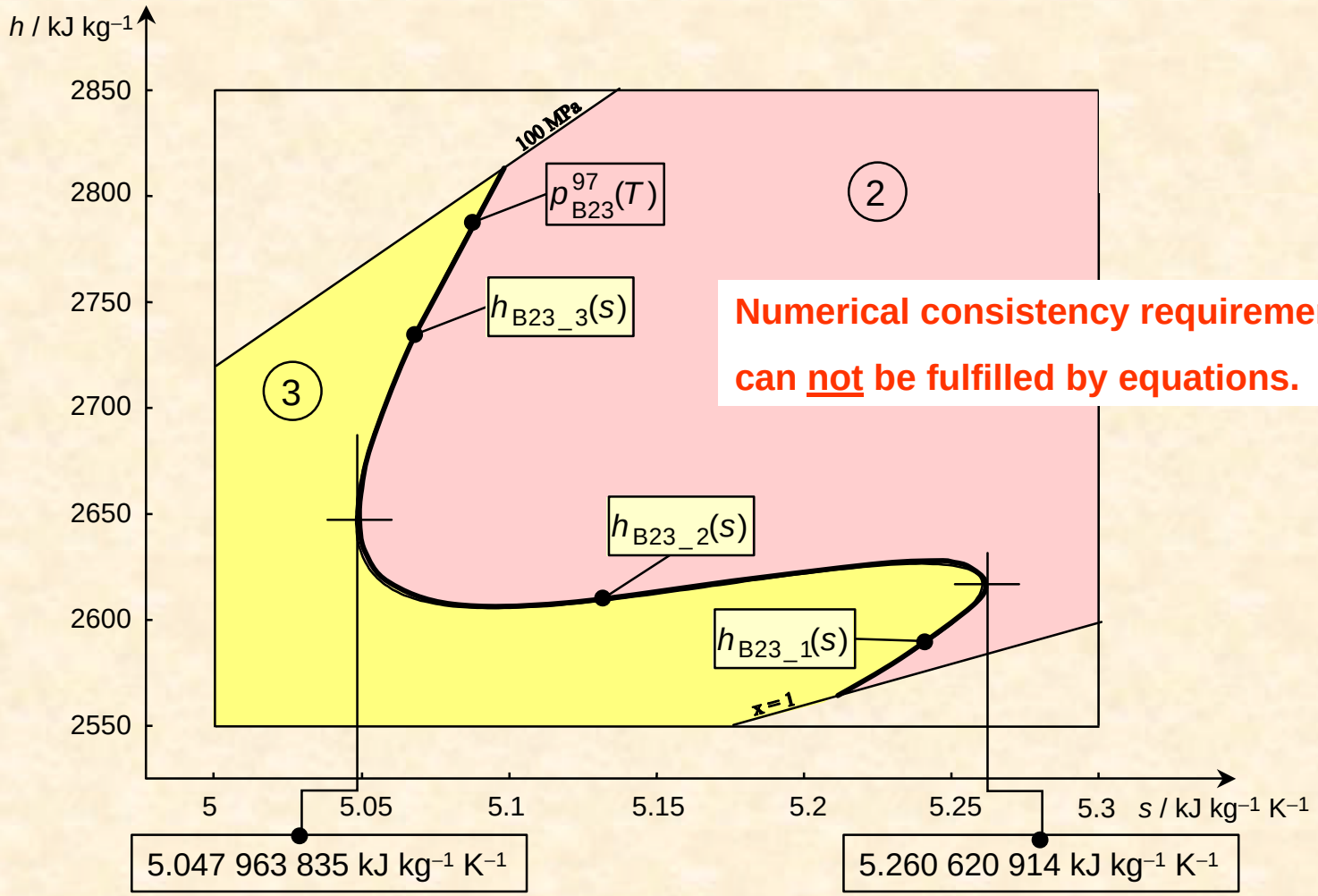
Numerical Consistencies of the Backward Equations with the Basic Equations of IAPWS-IF97 along the Boundary Equations

Equation	Region	$\lvert\Delta p\rvert_{tol}$	$\lvert\Delta p\rvert_{max}$	$\lvert\Delta T\rvert_{tol}$	$\lvert\Delta T\rvert_{max}$	$\lvert\Delta v/v\rvert_{tol}$	$\lvert\Delta v/v\rvert_{max}$
$h_{B13}(s)$	1	15 kPa	14.4 kPa	25 mK	24.2 mK	-	
	3	0.01 %	0.0039 %	25 mK	0.68 mK	0.01 %	0.0063 %

Equation	Region	$\lvert\Delta p\rvert_{tol}$	$\lvert\Delta p\rvert_{max}$	$\lvert\Delta T\rvert_{tol}$	$\lvert\Delta T\rvert_{max}$	$\lvert\Delta v/v\rvert_{tol}$	$\lvert\Delta v/v\rvert_{max}$
$h'(s)$	1	0.6 % 15 kPa	0.43 % 10.6 kPa	25 mK	23.8 mK	-	
	3	0.01 %	0.0066 %	25 mK	4.8 mK	0.01 %	0.0058 %

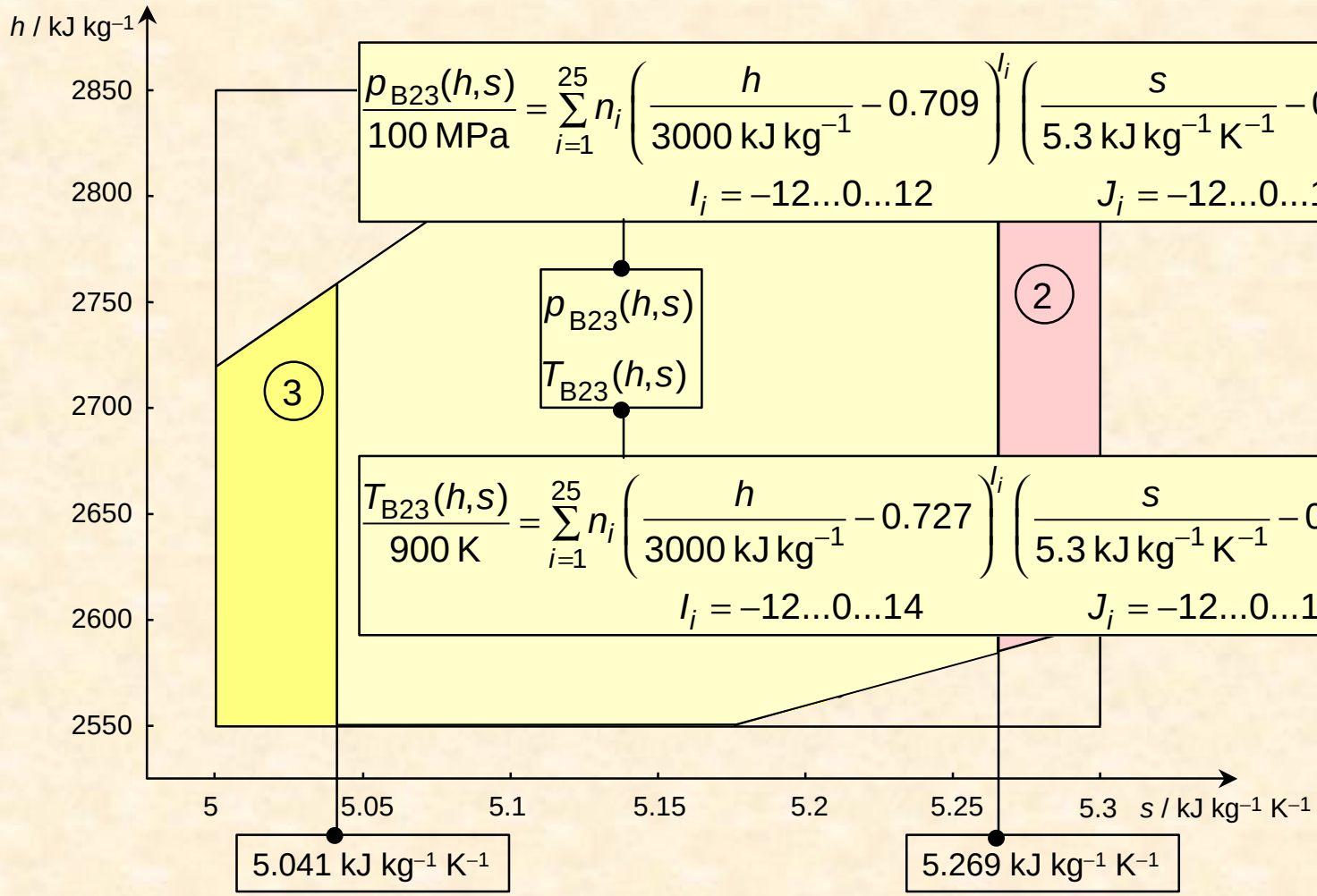
Equation	Region	$\lvert\Delta p/p\rvert_{tol}$	$\lvert\Delta p/p\rvert_{max}$	$\lvert\Delta T\rvert_{tol}$	$\lvert\Delta T\rvert_{max}$	$\lvert\Delta v/v\rvert_{tol}$	$\lvert\Delta v/v\rvert_{max}$
$h''(s)$	2	0.0035 % 0.0088 %	0.0028 % 0.0088 %	10 mK 25 mK	9.5 mK 23.9 mK	-	
	3	0.01 %	0.0056 %	25 mK	4.7 mK	0.01 %	0.0036 %

Boundary between Regions 2 and 3



Numerical consistency requirements can <u>not</u> be fulfilled by equations.

Equations for Boundary between Regions 2 and 3



$$\frac{p_{B23}(h,s)}{100\,\text{MPa}} = \sum_{i=1}^{25} n_i \left(\frac{h}{3000\,\text{kJ kg}^{-1}} - 0.709\right)^{I_i} \left(\frac{s}{5.3\,\text{kJ kg}^{-1}\,\text{K}^{-1}} - 0.824\right)^{J_i}$$

$$I_i = -12 \ldots 0 \ldots 12 \qquad J_i = -12 \ldots 0 \ldots 12$$

$$\frac{T_{B23}(h,s)}{900\,\text{K}} = \sum_{i=1}^{25} n_i \left(\frac{h}{3000\,\text{kJ kg}^{-1}} - 0.727\right)^{I_i} \left(\frac{s}{5.3\,\text{kJ kg}^{-1}\,\text{K}^{-1}} - 0.864\right)^{J_i}$$

$$I_i = -12 \ldots 0 \ldots 14 \qquad J_i = -12 \ldots 0 \ldots 10$$

Use of the Equation Set

(*h*,*s*)

$s < 5.041$ kJ kg^{-1} K^{-1} → **Region 3**

5.041 kJ kg^{-1} K^{-1} $\leq s \leq 5.269$ kJ kg^{-1} K^{-1} →

$$p_{B23} = p_{B23}(h,s)$$

$$T_{B23} = T_{B23}(h,s)$$

$$T_{B23}^{97} = T_{B23}^{97}(p_{B23})$$

$s > 5.269$ kJ kg^{-1} K^{-1} → **Region 2**

$$T_{B23} \leq T_{B23}^{97}$$

→ **Region 3**

$$T_{B23} > T_{B23}^{97}$$

→ **Region 2**

Numerical Consistency

Numerical Consistency with the B23-Equation of IAPWS-IF97

Equation	Region	$\lvert\Delta p / p\rvert_{tol}$	$\lvert\Delta p / p\rvert_{max}$
$p_{B23}(h,s)$	2	0.0088 %	0.0048 %
	3	0.01 %	0.0056 %

Equation	Region	$\lvert\Delta T\rvert_{tol}$	$\lvert\Delta T\rvert_{max}$
$T_{B23}(h,s)$	2	25 mK	5.0 mK
	3		18.8 mK

Numerical Consistency of the Backward Equations with the Basic Equations of IAPWS-IF97 along the B23-Boundary

Equations	Region	$\lvert\Delta p / p\rvert_{tol}$	$\lvert\Delta p / p\rvert_{max}$	$\lvert\Delta T\rvert_{tol}$	$\lvert\Delta T\rvert_{max}$	$\lvert\Delta v / v\rvert_{tol}$	$\lvert\Delta v / v\rvert_{max}$
$p_{B23}(h,s)$ $T_{B23}(h,s)$	2	0.0088 %	0.0029 %	25 mK	22.2 mK	-	
	3	0.01 %	0.0029 %	25 mK	19.4 mK	0.01 %	0.0080 %

Accuracy of the equations $p_{B23}(h,s)$ and $T_{B23}(h,s)$

is sufficient to determine the region for a given state point.

Computing Time in Relation to IAPWS-IF97

Determination of the region for a given (h,s) state point:

by iteration from IAPWS-IF97		by new equations	
Boundary	Computing Time	Computing Time	*CTR*
T= 623.15 K	4.5 μs/call	0.13 μs/call	34
saturated liquid line	5.9 μs/call	0.17 μs/call	34
saturated vapor line	9.8 μs/call	0.68 μs/call	14
$p_{\mathrm{B23}}^{97}(T)$	12.4 μs/call	0.62 μs/call	20

$$CTR = \frac{\text{Computing time using IAPWS-IF97 equations only}}{\text{Computing time using the equations for region boundaries}}$$

Conclusion - Equations for Region Boundaries

Equations as functions of *h* and *s* for region boundaries have been presented.

⇩

Accuracy of the equations is sufficient

to determine the region for a given (*h*,*s*) state point.

⇩

Iterations to determine the region from given *h* and *s* can be avoided.

⇩

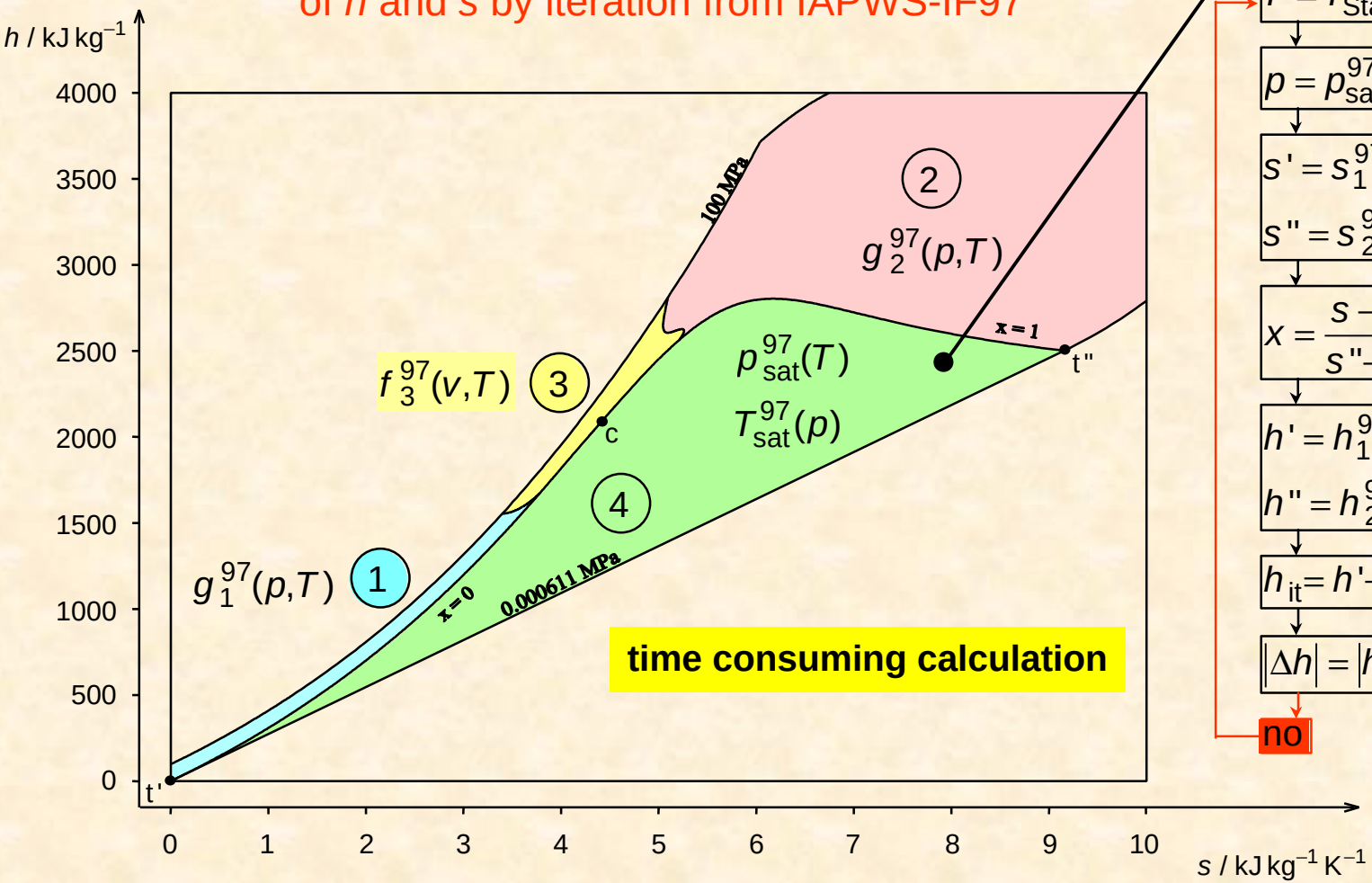
The determination of the region for a given (*h*,*s*) state point

is at least 14 times faster than IAPWS-IF97.

Iterations in the single-phase regions and along the region boundaries are replaced by explicit equations.

Calculation of Saturation Properties from Given *h* and *s*

Calculation of saturation properties as function of *h* and *s* by iteration from IAPWS-IF97



h,s

$T = T_{\text{Start}}$

$p = p_{\text{sat}}^{97}(T)$

$s' = s_1^{97}(p,T)$

$s'' = s_2^{97}(p,T)$

$x = \dfrac{s - s'}{s'' - s'}$

$h' = h_1^{97}(p,T)$

$h'' = h_2^{97}(p,T)$

$h_{\text{it}} = h' + x(h'' - h')$

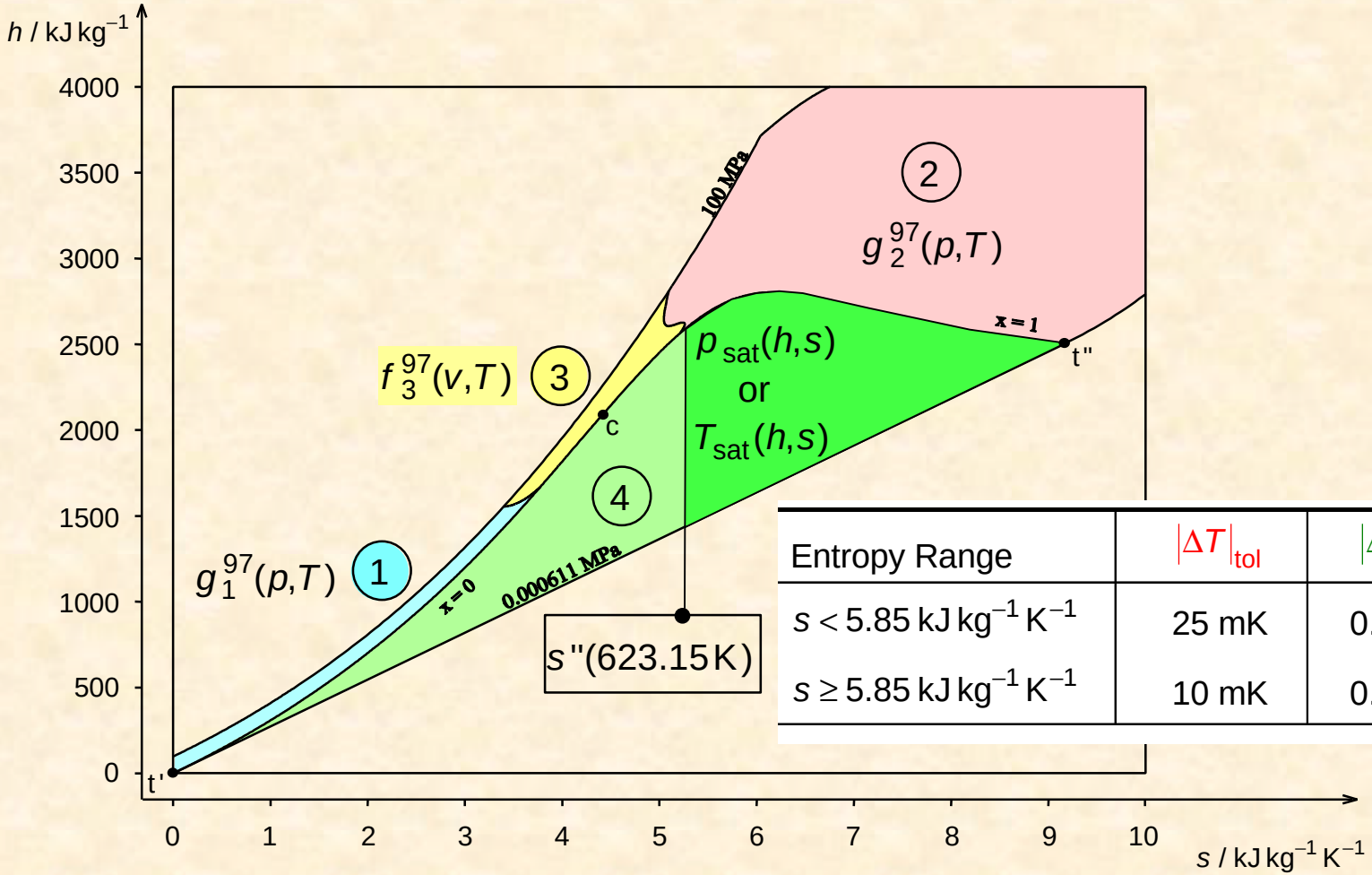
$|\Delta h| = |h_{\text{it}} - h| \le |\Delta h|_{\text{tol}}$

no

yes

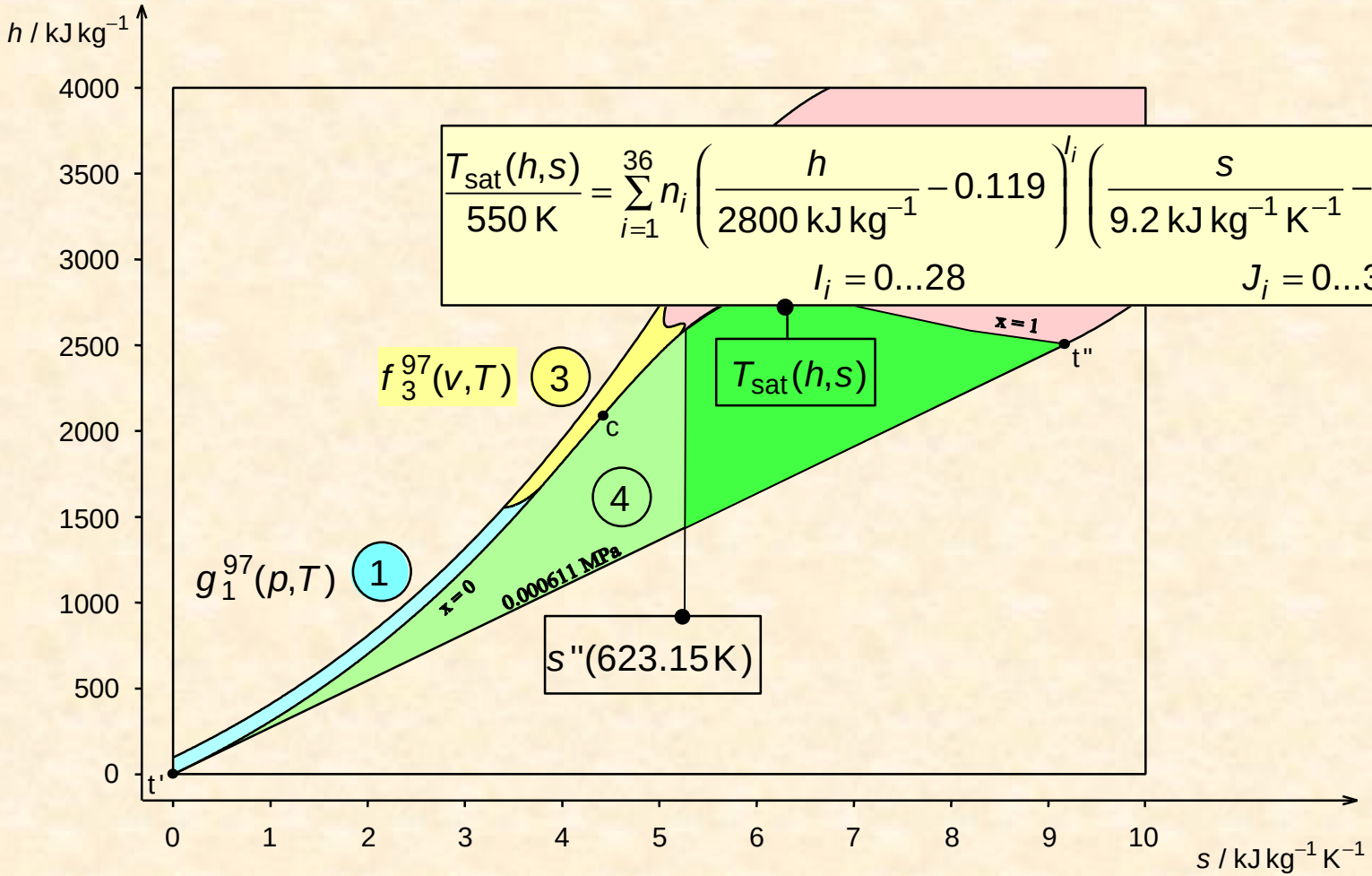
p,T,x

Numerical Consistency Requirements



Entropy Range	$\lvert\Delta T\rvert_{tol}$	$\lvert\Delta p/p\rvert_{tol}$
$s < 5.85\ \mathrm{kJ\,kg^{-1}\,K^{-1}}$	25 mK	0.0088 %
$s \geq 5.85\ \mathrm{kJ\,kg^{-1}\,K^{-1}}$	10 mK	0.0035 %

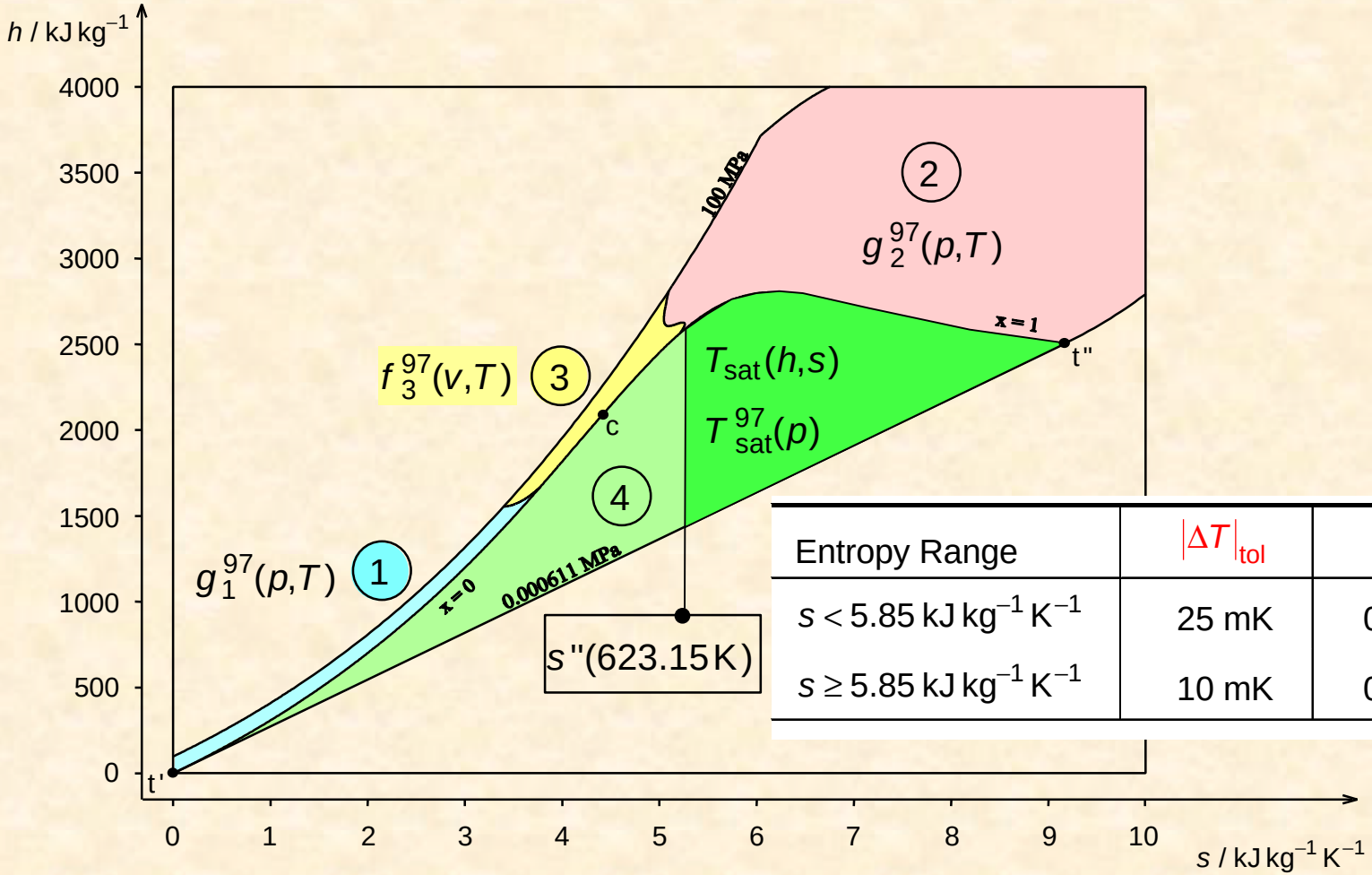
An Equation $T_{sat}(h,s)$ for Wet Steam



$$\frac{T_{sat}(h,s)}{550\,\mathrm{K}} = \sum_{i=1}^{36} n_i \left(\frac{h}{2800\,\mathrm{kJ\,kg^{-1}}} - 0.119\right)^{I_i} \left(\frac{s}{9.2\,\mathrm{kJ\,kg^{-1}\,K^{-1}}} - 1.07\right)^{J_i}$$

$I_i = 0...28$ $\qquad$ $J_i = 0...36$

Numerical Consistency with the Saturation-Temperature Equation of IAPWS-IF97



Entropy Range	$\lvert\Delta T\rvert_{tol}$	$\lvert\Delta T\rvert_{max}$
$s < 5.85\ \mathrm{kJ\,kg^{-1}\,K^{-1}}$	25 mK	0.67 mK
$s \geq 5.85\ \mathrm{kJ\,kg^{-1}\,K^{-1}}$	10 mK	0.86 mK

Calculation of the Functions $p_{sat}(h,s)$ and $x(h,s)$

Function $p_{sat}(h,s)$:

$$p_{sat}^{97}(T) \rightarrow p_{sat}(h,s)$$

$$T_{sat}(h,s) \rightarrow p_{sat}(h,s)$$

Numerical Consistency with the Saturation-Pressure Equation of IAPWS-IF97

Entropy Range	$\lvert\Delta p/p\rvert_{tol}$	$\lvert\Delta p/p\rvert_{max}$
$s < 5.85\ \mathrm{kJ\,kg^{-1}\,K^{-1}}$	0.0088 %	0.0029 %
$s \geq 5.85\ \mathrm{kJ\,kg^{-1}\,K^{-1}}$	0.0035 %	0.0034 %

Function $x(h,s)$:

$$T_{sat}(h,s) \rightarrow p_{sat}^{97}(T) \rightarrow \begin{aligned} h' &= h_1^{97}(p_{sat}, T_{sat}) \\ h'' &= h_2^{97}(p_{sat}, T_{sat}) \end{aligned} \rightarrow x = \frac{h - h'}{h'' - h'}$$

Numerical Consistency with the Basic Equation of IAPWS-IF97

Entropy Range	$\lvert\Delta x\rvert_{max}$
$s < 5.85\ \mathrm{kJ\,kg^{-1}\,K^{-1}}$	$0.65 \times 10^{-6}\ \mathrm{kg\,kg^{-1}}$
$s \geq 5.85\ \mathrm{kJ\,kg^{-1}\,K^{-1}}$	$4.40 \times 10^{-6}\ \mathrm{kg\,kg^{-1}}$

The accuracy of temperature calculated by $T_{sat}(h,s)$ is sufficient for calculating $p_{sat}(h,s)$ and $x(h,s)$.

Computing Time in Relation to IAPWS-IF97

Calculation of the functions T, p and $x(h,s)$:

by iteration from IAPWS-IF97	by new equation
Computing Time	Computing Time
15 μs/call	1.4 μs/call

$$CTR = \frac{\text{Computing time using IAPWS-IF97 equations only}}{\text{Computing time using the backward equation}} = 11$$

Conclusion - Backward Equation $T_{sat}(h,s)$ for Wet Steam

Backward equation $T_{sat}(h,s)$ for wet steam has been presented.

Functions $p_{sat}(h,s)$ and $x(h,s)$ can be calculated using

the saturation-pressure equation and the basic equations of IAPWS-IF97 .

⇩

Numerical Consistency is sufficient for process modeling

⇩

Iterations in the wet steam region can be avoided

⇩

The calculation of T_{sat}, p_{sat} and $x(h,s)$ is 11 times faster than IAPWS-IF97.

Summary

Backward equations $p(h,s)$ for region 3,
equations as functions of h and s for the region boundaries,
and an equation $T_{sat}(h,s)$ for wet steam
have been presented.

⇩

Presented equations are ready for evaluation by IAPWS.

⇩

Supplementary Release to the IAPWS-IF97