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Revised Supplementary Release on Backward Equations for the Functions $T(p,h)$, $v(p,h)$ and $T(p,s)$, $v(p,s)$ for Region 3 of the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam

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President:

Professor Tamara Petrova
Moscow Engineering Power Institute
Moscow, Russia

Executive Secretary:

Dr. R. B. Dooley
Structural Integrity Associates
Southport, Merseyside, UK
Email: bdooley@structint.com

This revised supplementary release replaces the corresponding revised supplementary release of 2004, and contains 22 pages, including this cover page.

This revised supplementary release has been authorized by the International Association for the Properties of Water and Steam (IAPWS) at its meeting in Moscow, Russia, 22-27 June, 2014, for issue by its Secretariat. The members of IAPWS are: Britain and Ireland, Canada, the Czech Republic, Germany, Japan, Russia, Scandinavia (Denmark, Finland, Norway, Sweden), and the United States, and associate members Argentina & Brazil, Australia, France, Greece, Italy, New Zealand, and Switzerland.

The backward equations for temperature and specific volume as functions of pressure and enthalpy $T(p,h)$, $v(p,h)$ and as functions of pressure and entropy $T(p,s)$, $v(p,s)$ for region 3, and the equations for saturation pressure as a function of enthalpy $p_{3\text{sat}}(h)$ and as a function of entropy $p_{3\text{sat}}(s)$ for the saturation boundaries of region 3 provided in this release are recommended as a supplement to "IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam" (IAPWS-IF97) [1, 2]. Further details concerning the equations can be found in the corresponding article by H.-J. Kretzschmar et al. [3].

This revision consists of edits to clarify descriptions of how to determine the region or subregion; the property calculations are unchanged.

Further information concerning this supplementary release, other releases, supplementary releases, guidelines, technical guidance documents, and advisory notes issued by IAPWS can be obtained from the Executive Secretary of IAPWS or from <http://www.iapws.org>.

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1 Nomenclature

Thermodynamic quantities:

f	Specific Helmholtz free energy
h	Specific enthalpy
p	Pressure
s	Specific entropy
T	Absolute temperature ^a
v	Specific volume
Δ	Difference in any quantity
η	Reduced enthalpy, $\eta = h/h^*$
θ	Reduced temperature $\theta = T/T^*$
π	Reduced pressure, $\pi = p/p^*$
ρ	Density
σ	Reduced entropy, $\sigma = s/s^*$
ω	Reduced volume, $\omega = v/v^*$
x	Vapor fraction

Root-mean-square value:

$$\Delta x_{\text{RMS}} = \sqrt{\frac{1}{N} \sum_{n=1}^N (\Delta x_n)^2}$$

where Δx_n can be either absolute or percentage difference between the corresponding quantities x ; N is the number of Δx_n values (100 million points uniformly distributed over the range of validity in the p - T plane).

Superscripts:

01	Equation of IAPWS-IF97-S01
97	Quantity or equation of IAPWS-IF97
*	Reducing quantity
'	Saturated liquid state
"	Saturated vapor state

Subscripts:

1	Region 1
2	Region 2
3	Region 3
3a	Subregion 3a
3b	Subregion 3b
3ab	Boundary between subregions 3a and 3b
4	Region 4
5	Region 5
B23	Boundary between regions 2 and 3
c	Critical point
it	Iterated quantity
max	Maximum value of a quantity
RMS	Root-mean-square value of a quantity
sat	Saturation state
tol	Tolerance, range of accepted value of a quantity

^a Note: T denotes absolute temperature on the International Temperature Scale of 1990 (ITS-90).

2 Background

The Industrial Formulation IAPWS-IF97 for the thermodynamic properties of water and steam [1, 2] contains basic equations, saturation equations and equations for the most often used backward functions $T(p, h)$ and $T(p, s)$ valid in the liquid region 1 and the vapor region 2; see Figure 1. The IAPWS-IF97 was supplemented by "Backward Equations for Pressure as a Function of Enthalpy and Entropy $p(h, s)$ to the Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam" [4, 5], which is referred to here as IAPWS-IF97-S01, including equations for the backward function $p(h, s)$ valid in region 1 and region 2.

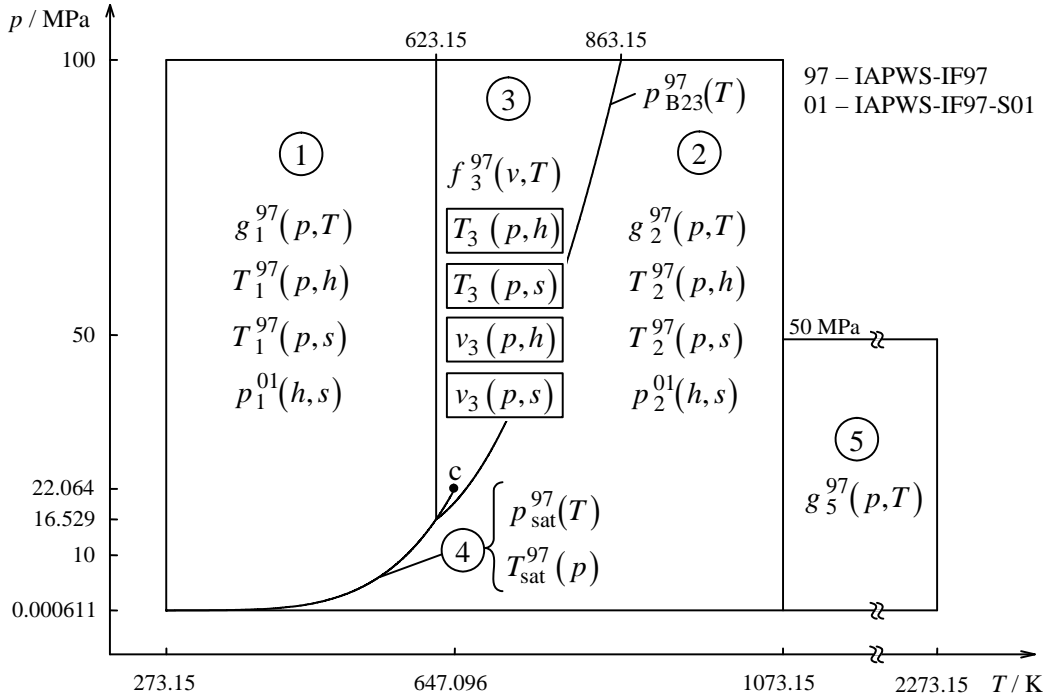


Figure 1. Regions and equations of IAPWS-IF97, IAPWS-IF97-S01, and the backward equations $T(p, h)$, $v(p, h)$, and $T(p, s)$, $v(p, s)$ of this release

In modeling steam power cycles, thermodynamic properties as functions of the variables (p, h) or (p, s) are also required in region 3. It is difficult to perform these calculations with IAPWS-IF97, because two-dimensional iteration is required using the functions $p(v, T)$, $h(v, T)$ or $p(v, T)$, $s(v, T)$ that can be explicitly calculated from the fundamental region 3 equation $f(v, T)$. While these calculations are not frequently required in region 3, the relatively large computing time required for two-dimensional iteration can be significant in process modeling.

In order to avoid such iterations, this release provides equations for the backward functions $T_3(p, h)$, $v_3(p, h)$ and $T_3(p, s)$, $v_3(p, s)$, see Figure 1. With temperature and specific

volume calculated from the backward equations, the other properties in region 3 can be calculated using the IAPWS-IF97 basic equation $f_3^{97}(v, T)$.

In addition, boundary equations for the saturation pressure as a function of enthalpy $p_{3\text{sat}}(h)$ and as a function of entropy $p_{3\text{sat}}(s)$ for the saturated liquid and vapor lines of region 3 are provided. Using these equations, whether a state point is located in the single-phase region or in the two-phase (wet steam) region can be determined without iteration. Section 4 contains the comprehensive description of the boundary equations.

The numerical consistencies of all backward equations and boundary equations presented in Sections 3 and 4 with the IAPWS-IF97 basic equation are sufficient for most applications in heat cycle and steam turbine calculations. For applications where the demands on numerical consistency are extremely high, iterations using the IAPWS-IF97 basic equation may be necessary. In these cases, the backward or boundary equations can be used for calculating very accurate starting values. The time required to reach the convergence criteria of the iteration will be significantly reduced.

The presented backward and boundary equations can only be used in their ranges of validity described in Sections 3.2, 4.3, and 4.4. They should not be used for determining any thermodynamic derivatives.

In any case, depending on the application, a conscious decision is required whether to use the backward or boundary equations or to calculate the corresponding values by iterations from the basic equation of IAPWS-IF97.

3 Backward Equations $T(p, h)$, $v(p, h)$, $T(p, s)$, and $v(p, s)$ for Region 3

3.1 Numerical Consistency Requirements

The permissible value for the numerical consistency $|\Delta T|_{\text{tol}} = 25 \text{ mK}$ of the backward functions $T_3(p, h)$ and $T_3(p, s)$ with the basic equation $f_3^{97}(v, T)$ was determined by IAPWS [6, 7] as a result of an international survey.

The permissible value Δv_{tol} for the numerical consistency for the equations $v_3(p, h)$ and $v_3(p, s)$ can be estimated from the total differentials

$$\Delta v_{\text{tol}} = \left(\frac{\partial v}{\partial T} \right)_h \Delta T_{\text{tol}} + \left(\frac{\partial v}{\partial h} \right)_T \Delta h_{\text{tol}} \quad \text{and} \quad \Delta v_{\text{tol}} = \left(\frac{\partial v}{\partial T} \right)_s \Delta T_{\text{tol}} + \left(\frac{\partial v}{\partial s} \right)_T \Delta s_{\text{tol}} ,$$

where $\left(\frac{\partial v}{\partial T} \right)_h$, $\left(\frac{\partial v}{\partial h} \right)_T$, $\left(\frac{\partial v}{\partial T} \right)_s$, and $\left(\frac{\partial v}{\partial s} \right)_T$ are derivatives [8] calculated from the IAPWS-IF97 basic equation and Δh_{tol} and Δs_{tol} are values determined by IAPWS for the adjacent

region 1 and subregion 2c [9], see Table 1. The resulting permissible specific volume difference is $|\Delta v/v|_{\text{tol}} = 0.01\%$ for both functions $v_3(p, h)$ and $v_3(p, s)$.

At the critical point $\left[T_c = 647.096 \text{ K}, v_c = 1/(322 \text{ kg m}^{-3}) \right]$ [10], more stringent consistency requirements were arbitrarily set. These were $|\Delta T|_{\text{tol}} = 0.49 \text{ mK}$ and $|\Delta v/v|_{\text{tol}} = 0.0001\%$.

Table 1. Numerical consistency values $|\Delta T|_{\text{tol}}$ of [6] required for $T_3(p, h)$ and $T_3(p, s)$, values $|\Delta h|_{\text{tol}}$, $|\Delta s|_{\text{tol}}$ of [9], and resulting tolerances $|\Delta v/v|_{\text{tol}}$ required for $v_3(p, h)$ and $v_3(p, s)$

	$ \Delta T _{\text{tol}}$	$ \Delta h _{\text{tol}}$	$ \Delta s _{\text{tol}}$	$ \Delta v/v _{\text{tol}}$
Region 3	25 mK	80 J kg ⁻¹	0.1 J kg ⁻¹ K ⁻¹	0.01 %
Critical Point	0.49 mK	-	-	0.0001 %

3.2 Structure of the Equation Set

The equation set consists of backward equations $T(p, h)$, $v(p, h)$ and $T(p, s)$, $v(p, s)$ for region 3. Region 3 is defined by:

$$623.15 \text{ K} \leq T \leq 863.15 \text{ K} \quad \text{and} \quad p_{\text{B23}}^{97}(T) \leq p \leq 100 \text{ MPa},$$

where p_{B23}^{97} represents the B23 equation of IAPWS-IF97. Figure 2 shows the way in which region 3 is divided into the two subregions 3a and 3b.

The boundary between the subregions 3a and 3b corresponds to the critical isentropic line

$$s = s_c = 4.412\,021\,482\,234\,76 \text{ kJ kg}^{-1} \text{ K}^{-1};$$

see Figure 2. For the functions $T(p, s)$ and $v(p, s)$, input points can be tested directly to identify the subregion since the specific entropy is an independent variable.

In order to decide which $T(p, h)$, $v(p, h)$ equation, 3a or 3b, must be used for given values of p and h , the boundary equation $h_{3\text{ab}}(p)$, Eq. (1), has to be used; see Figure 2. This equation is a polynomial of the third degree and reads

$$\frac{h_{3\text{ab}}(p)}{h^*} = \eta(\pi) = n_1 + n_2\pi + n_3\pi^2 + n_4\pi^3, \quad (1)$$

where $\eta = h/h^*$ and $\pi = p/p^*$ with $h^* = 1 \text{ kJ kg}^{-1}$ and $p^* = 1 \text{ MPa}$. The coefficients n_1 to n_4 of Eq. (1) are listed in Table 2. The range of the equation $h_{3\text{ab}}(p)$ is from the critical point to 100 MPa. The related temperature at 100 MPa is $T = 762.380\,873\,481 \text{ K}$. Equation (1) does not exactly describe the critical isentropic line.

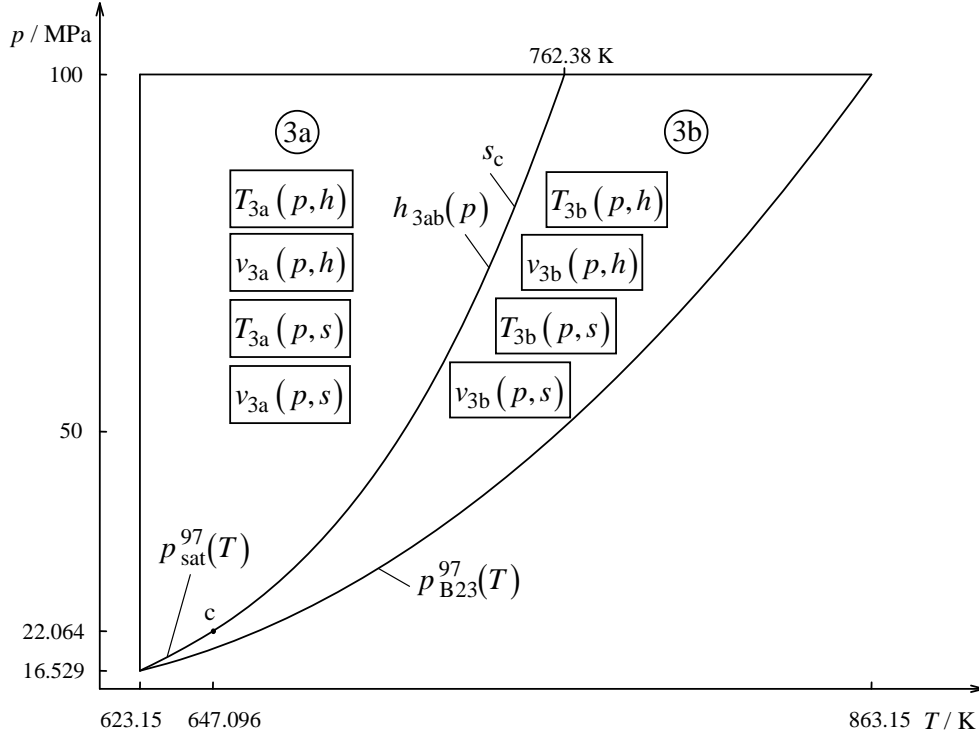


Figure 2. Division of region 3 into two subregions 3a and 3b for the backward equations $T(p,h)$, $v(p,h)$ and $T(p,s)$, $v(p,s)$

Table 2. Numerical values of the coefficients of the equation $h_{3ab}(p)$ in its dimensionless form, Eq. (1), for defining the boundary between subregions 3a and 3b

i	n_i	i	n_i
1	$0.201\ 464\ 004\ 206\ 875 \times 10^4$	3	$-0.219\ 921\ 901\ 054\ 187 \times 10^{-1}$
2	$0.374\ 696\ 550\ 136\ 983 \times 10^1$	4	$0.875\ 131\ 686\ 009\ 950 \times 10^{-4}$

The maximum specific entropy deviation was determined as

$$|\Delta s_{3ab}|_{\max} = \left| s_3^{97}(T_{\text{it}}^{97}(p, h_{3ab}(p)), v_{\text{it}}^{97}(p, h_{3ab}(p))) - s_c \right|_{\max} = 0.66 \text{ J kg}^{-1} \text{ K}^{-1},$$

where T_{it}^{97} and v_{it}^{97} were obtained by iterations using the derivatives $p_3^{97}(v, T)$ and $s_3^{97}(v, T)$ of the IAPWS-IF97 basic equation for region 3.

Equation (1) does not correctly reproduce the isentropic line $s = s_c$ at pressures lower than p_c . However, the calculated values $h_{3ab}(p)$ are not higher than the enthalpy on the saturated vapor line and not lower than the enthalpy on the saturated liquid line.

For *computer-program verification*, Eq. (1) gives the following p - h point:

$$p = 25 \text{ MPa} , h_{3ab}(p) = 2.095\,936\,454 \times 10^3 \text{ kJ kg}^{-1} .$$

3.3 Backward Equations $T(p,h)$ and $v(p,h)$ for Subregions 3a and 3b

The Equations $T(p,h)$. The backward equation $T_{3a}(p,h)$ for subregion 3a has the following dimensionless form:

$$\frac{T_{3a}(p,h)}{T^*} = \theta_{3a}(\pi,\eta) = \sum_{i=1}^{31} n_i (\pi + 0.240)^{I_i} (\eta - 0.615)^{J_i} , \quad (2)$$

where $\theta = T/T^*$, $\pi = p/p^*$, and $\eta = h/h^*$, with $T^* = 760 \text{ K}$, $p^* = 100 \text{ MPa}$, and $h^* = 2300 \text{ kJ kg}^{-1}$. The coefficients n_i and exponents I_i and J_i of Eq. (2) are listed in Table 3.

The backward equation $T_{3b}(p,h)$ for subregion 3b reads in its dimensionless form

$$\frac{T_{3b}(p,h)}{T^*} = \theta_{3b}(\pi,\eta) = \sum_{i=1}^{33} n_i (\pi + 0.298)^{I_i} (\eta - 0.720)^{J_i} , \quad (3)$$

where $\theta = T/T^*$, $\pi = p/p^*$, and $\eta = h/h^*$, with $T^* = 860 \text{ K}$, $p^* = 100 \text{ MPa}$, and $h^* = 2800 \text{ kJ kg}^{-1}$. The coefficients n_i and exponents I_i and J_i of Eq. (3) are listed in Table 4.

Table 3. Coefficients and exponents of the backward equation $T_{3a}(p,h)$ for subregion 3a in its dimensionless form, Eq. (2)

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	-12	0	-0.133 645 667 811 215 $\times 10^{-6}$	17	-3	0	-0.384 460 997 596 657 $\times 10^{-5}$
2	-12	1	0.455 912 656 802 978 $\times 10^{-5}$	18	-2	1	0.337 423 807 911 655 $\times 10^{-2}$
3	-12	2	-0.146 294 640 700 979 $\times 10^{-4}$	19	-2	3	-0.551 624 873 066 791
4	-12	6	0.639 341 312 970 080 $\times 10^{-2}$	20	-2	4	0.729 202 277 107 470
5	-12	14	0.372 783 927 268 847 $\times 10^3$	21	-1	0	-0.992 522 757 376 041 $\times 10^{-2}$
6	-12	16	-0.718 654 377 460 447 $\times 10^4$	22	-1	2	-0.119 308 831 407 288
7	-12	20	0.573 494 752 103 400 $\times 10^6$	23	0	0	0.793 929 190 615 421
8	-12	22	-0.267 569 329 111 439 $\times 10^7$	24	0	1	0.454 270 731 799 386
9	-10	1	-0.334 066 283 302 614 $\times 10^{-4}$	25	1	1	0.209 998 591 259 910
10	-10	5	-0.245 479 214 069 597 $\times 10^{-1}$	26	3	0	-0.642 109 823 904 738 $\times 10^{-2}$
11	-10	12	0.478 087 847 764 996 $\times 10^2$	27	3	1	-0.235 155 868 604 540 $\times 10^{-1}$
12	-8	0	0.764 664 131 818 904 $\times 10^{-5}$	28	4	0	0.252 233 108 341 612 $\times 10^{-2}$
13	-8	2	0.128 350 627 676 972 $\times 10^{-2}$	29	4	3	-0.764 885 133 368 119 $\times 10^{-2}$
14	-8	4	0.171 219 081 377 331 $\times 10^{-1}$	30	10	4	0.136 176 427 574 291 $\times 10^{-1}$
15	-8	10	-0.851 007 304 583 213 $\times 10^1$	31	12	5	-0.133 027 883 575 669 $\times 10^{-1}$
16	-5	2	-0.136 513 461 629 781 $\times 10^{-1}$				

Table 4. Coefficients and exponents of the backward equation $T_{3b}(p, h)$ for subregion 3b in its dimensionless form, Eq. (3)

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	-12	0	$0.323\ 254\ 573\ 644\ 920 \times 10^{-4}$	18	-3	5	$-0.307\ 622\ 221\ 350\ 501 \times 10^1$
2	-12	1	$-0.127\ 575\ 556\ 587\ 181 \times 10^{-3}$	19	-2	0	$-0.574\ 011\ 959\ 864\ 879 \times 10^{-1}$
3	-10	0	$-0.475\ 851\ 877\ 356\ 068 \times 10^{-3}$	20	-2	4	$0.503\ 471\ 360\ 939\ 849 \times 10^1$
4	-10	1	$0.156\ 183\ 014\ 181\ 602 \times 10^{-2}$	21	-1	2	$-0.925\ 081\ 888\ 584\ 834$
5	-10	5	$0.105\ 724\ 860\ 113\ 781$	22	-1	4	$0.391\ 733\ 882\ 917\ 546 \times 10^1$
6	-10	10	$-0.858\ 514\ 221\ 132\ 534 \times 10^2$	23	-1	6	$-0.773\ 146\ 007\ 130\ 190 \times 10^2$
7	-10	12	$0.724\ 140\ 095\ 480\ 911 \times 10^3$	24	-1	10	$0.949\ 308\ 762\ 098\ 587 \times 10^4$
8	-8	0	$0.296\ 475\ 810\ 273\ 257 \times 10^{-2}$	25	-1	14	$-0.141\ 043\ 719\ 679\ 409 \times 10^7$
9	-8	1	$-0.592\ 721\ 983\ 365\ 988 \times 10^{-2}$	26	-1	16	$0.849\ 166\ 230\ 819\ 026 \times 10^7$
10	-8	2	$-0.126\ 305\ 422\ 818\ 666 \times 10^{-1}$	27	0	0	$0.861\ 095\ 729\ 446\ 704$
11	-8	4	$-0.115\ 716\ 196\ 364\ 853$	28	0	2	$0.323\ 346\ 442\ 811\ 720$
12	-8	10	$0.849\ 000\ 969\ 739\ 595 \times 10^2$	29	1	1	$0.873\ 281\ 936\ 020\ 439$
13	-6	0	$-0.108\ 602\ 260\ 086\ 615 \times 10^{-1}$	30	3	1	$-0.436\ 653\ 048\ 526\ 683$
14	-6	1	$0.154\ 304\ 475\ 328\ 851 \times 10^{-1}$	31	5	1	$0.286\ 596\ 714\ 529\ 479$
15	-6	2	$0.750\ 455\ 441\ 524\ 466 \times 10^{-1}$	32	6	1	$-0.131\ 778\ 331\ 276\ 228$
16	-4	0	$0.252\ 520\ 973\ 612\ 982 \times 10^{-1}$	33	8	1	$0.676\ 682\ 064\ 330\ 275 \times 10^{-2}$
17	-4	1	$-0.602\ 507\ 901\ 232\ 996 \times 10^{-1}$				

Computer-program verification. To assist the user in computer-program verification of Eqs. (2) and (3), Table 5 contains test values for calculated temperatures.

Table 5. Selected temperature values calculated from Eqs. (2) and (3) ^a

Equation	p / MPa	h / kJ kg^{-1}	T / K
$T_{3a}(p, h)$, Eq. (2)	20	1700	$6.293\ 083\ 892 \times 10^2$
	50	2000	$6.905\ 718\ 338 \times 10^2$
	100	2100	$7.336\ 163\ 014 \times 10^2$
$T_{3b}(p, h)$, Eq. (3)	20	2500	$6.418\ 418\ 053 \times 10^2$
	50	2400	$7.351\ 848\ 618 \times 10^2$
	100	2700	$8.420\ 460\ 876 \times 10^2$

^a It is recommended that programmed functions be verified using 8 byte real values for all variables.

The Equations $v(p, h)$. The backward equation $v_{3a}(p, h)$ for subregion 3a has the following dimensionless form:

$$\frac{v_{3a}(p, h)}{v^*} = \omega_{3a}(\pi, \eta) = \sum_{i=1}^{32} n_i (\pi + 0.128)^{I_i} (\eta - 0.727)^{J_i}, \quad (4)$$

where $\omega = v/v^*$, $\pi = p/p^*$, and $\eta = h/h^*$, with $v^* = 0.0028 \text{ m}^3 \text{ kg}^{-1}$, $p^* = 100 \text{ MPa}$, and $h^* = 2100 \text{ kJ kg}^{-1}$. The coefficients n_i and exponents I_i and J_i of Eq. (4) are listed in Table 6.

The backward equation $v_{3b}(p, h)$ for subregion 3b reads in its dimensionless form

$$\frac{v_{3b}(p, h)}{v^*} = \omega_{3b}(\pi, \eta) = \sum_{i=1}^{30} n_i (\pi + 0.0661)^{I_i} (\eta - 0.720)^{J_i}, \quad (5)$$

where $\omega = v/v^*$, $\pi = p/p^*$, and $\eta = h/h^*$, with $v^* = 0.0088 \text{ m}^3 \text{ kg}^{-1}$, $p^* = 100 \text{ MPa}$, and $h^* = 2800 \text{ kJ kg}^{-1}$. The coefficients n_i and exponents I_i and J_i of Eq. (5) are listed in Table 7.

Table 6. Coefficients and exponents of the backward equation $v_{3a}(p, h)$ for subregion 3a in its dimensionless form, Eq. (4)

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	-12	6	0.529 944 062 966 028 $\times 10^{-2}$	17	-2	16	0.568 366 875 815 960 $\times 10^4$
2	-12	8	-0.170 099 690 234 461	18	-1	0	0.808 169 540 124 668 $\times 10^{-2}$
3	-12	12	0.111 323 814 312 927 $\times 10^2$	19	-1	1	0.172 416 341 519 307
4	-12	18	-0.217 898 123 145 125 $\times 10^4$	20	-1	2	0.104 270 175 292 927 $\times 10^1$
5	-10	4	-0.506 061 827 980 875 $\times 10^{-3}$	21	-1	3	-0.297 691 372 792 847
6	-10	7	0.556 495 239 685 324	22	0	0	0.560 394 465 163 593
7	-10	10	-0.943 672 726 094 016 $\times 10^1$	23	0	1	0.275 234 661 176 914
8	-8	5	-0.297 856 807 561 527	24	1	0	-0.148 347 894 866 012
9	-8	12	0.939 353 943 717 186 $\times 10^2$	25	1	1	-0.651 142 513 478 515 $\times 10^{-1}$
10	-6	3	0.192 944 939 465 981 $\times 10^{-1}$	26	1	2	-0.292 468 715 386 302 $\times 10^1$
11	-6	4	0.421 740 664 704 763	27	2	0	0.664 876 096 952 665 $\times 10^{-1}$
12	-6	22	-0.368 914 126 282 330 $\times 10^7$	28	2	2	0.352 335 014 263 844 $\times 10^1$
13	-4	2	-0.737 566 847 600 639 $\times 10^{-2}$	29	3	0	-0.146 340 792 313 332 $\times 10^{-1}$
14	-4	3	-0.354 753 242 424 366	30	4	2	-0.224 503 486 668 184 $\times 10^1$
15	-3	7	-0.199 768 169 338 727 $\times 10^1$	31	5	2	0.110 533 464 706 142 $\times 10^1$
16	-2	3	0.115 456 297 059 049 $\times 10^1$	32	8	2	-0.408 757 344 495 612 $\times 10^{-1}$

Table 7. Coefficients and exponents of the backward equation $v_{3b}(p, h)$ for subregion 3b in its dimensionless form, Eq. (5)

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	-12	0	-0.225 196 934 336 318 $\times 10^{-8}$	16	-4	6	-0.321 087 965 668 917 $\times 10^1$
2	-12	1	0.140 674 363 313 486 $\times 10^{-7}$	17	-4	10	0.607 567 815 637 771 $\times 10^3$
3	-8	0	0.233 784 085 280 560 $\times 10^{-5}$	18	-3	0	0.557 686 450 685 932 $\times 10^{-3}$
4	-8	1	-0.331 833 715 229 001 $\times 10^{-4}$	19	-3	2	0.187 499 040 029 550
5	-8	3	0.107 956 778 514 318 $\times 10^{-2}$	20	-2	1	0.905 368 030 448 107 $\times 10^{-2}$
6	-8	6	-0.271 382 067 378 863	21	-2	2	0.285 417 173 048 685
7	-8	7	0.107 202 262 490 333 $\times 10^1$	22	-1	0	0.329 924 030 996 098 $\times 10^{-1}$
8	-8	8	-0.853 821 329 075 382	23	-1	1	0.239 897 419 685 483
9	-6	0	-0.215 214 194 340 526 $\times 10^{-4}$	24	-1	4	0.482 754 995 951 394 $\times 10^1$
10	-6	1	0.769 656 088 222 730 $\times 10^{-3}$	25	-1	5	-0.118 035 753 702 231 $\times 10^2$
11	-6	2	-0.431 136 580 433 864 $\times 10^{-2}$	26	0	0	0.169 490 044 091 791
12	-6	5	0.453 342 167 309 331	27	1	0	-0.179 967 222 507 787 $\times 10^{-1}$
13	-6	6	-0.507 749 535 873 652	28	1	1	0.371 810 116 332 674 $\times 10^{-1}$
14	-6	10	-0.100 475 154 528 389 $\times 10^3$	29	2	2	-0.536 288 335 065 096 $\times 10^{-1}$
15	-4	3	-0.219 201 924 648 793	30	2	6	0.160 697 101 092 520 $\times 10^1$

Computer-program verification. To assist the user in computer-program verification of Eqs. (4) and (5), Table 8 contains test values for calculated specific volumes.

Table 8. Selected specific volume values calculated from Eqs. (4) and (5) ^a

Equation	p / MPa	h / kJ kg ⁻¹	v / m ³ kg ⁻¹
$v_{3a}(p, h)$, Eq. (4)	20	1700	$1.749\ 903\ 962 \times 10^{-3}$
	50	2000	$1.908\ 139\ 035 \times 10^{-3}$
	100	2100	$1.676\ 229\ 776 \times 10^{-3}$
$v_{3b}(p, h)$, Eq. (5)	20	2500	$6.670\ 547\ 043 \times 10^{-3}$
	50	2400	$2.801\ 244\ 590 \times 10^{-3}$
	100	2700	$2.404\ 234\ 998 \times 10^{-3}$

^a It is recommended that programmed functions be verified using 8 byte real values for all variables.

Numerical Consistency with the Basic Equation of IAPWS-IF97. The maximum temperature differences and related root-mean-square differences between the calculated temperature Eqs. (2) and (3) and the IAPWS-IF97 basic equation $f_3^{97}(v, T)$ in comparison with the permissible differences are listed in Table 9. The calculation of the root-mean-square values is described in Section 1.

Table 9 also contains the maximum relative deviations and root-mean-square relative deviations for specific volume of Eqs. (4) and (5) from IAPWS-IF97.

The critical temperature and the critical volume are met exactly by the equations $T(p, h)$ and $v(p, h)$.

Table 9. Maximum differences and root-mean-square differences of the temperature calculated from Eqs. (2) and (3) and specific volume calculated from Eqs. (4) and (5) to the IAPWS-IF97 basic equation $f_3^{97}(v, T)$ and related permissible values

Subregion	Equation	$ \Delta T _{\text{tol}}$	$ \Delta T _{\text{max}}$	$ \Delta T _{\text{RMS}}$
3a	(2)	25 mK	23.6 mK	10.5 mK
3b	(3)	25 mK	19.6 mK	9.6 mK
Subregion	Equation	$ \Delta v/v _{\text{tol}}$	$ \Delta v/v _{\text{max}}$	$ \Delta v/v _{\text{RMS}}$
3a	(4)	0.01 %	0.0080 %	0.0032 %
3b	(5)	0.01 %	0.0095 %	0.0042 %

Consistency at Boundary Between Subregions. The maximum temperature difference between the two backward equations, Eq. (2) and Eq. (3), along the boundary $h_{3ab}(p)$, Eq. (1), has the following value

$$|\Delta T|_{\text{max}} = \left| T_{3a}(p, h_{3ab}(p)) - T_{3b}(p, h_{3ab}(p)) \right|_{\text{max}} = 0.37 \text{ mK}.$$

Thus, the temperature differences between the two backward functions $T(p, h)$ of the adjacent subregions are smaller than the numerical consistencies with the IAPWS-IF97 equations.

The relative specific volume differences between the two backward equations $v(p, h)$ of the adjacent subregions 3a and 3b are also smaller than the numerical consistencies of these equations with the IAPWS-IF97 basic equation. Along the boundary $h_{3ab}(p)$, Eq. (1), the maximum difference between the corresponding equations was determined as:

$$\left| \frac{\Delta v}{v} \right|_{\max} = \left| \frac{v_{3a}(p, h_{3ab}(p)) - v_{3b}(p, h_{3ab}(p))}{v_{3b}(p, h_{3ab}(p))} \right|_{\max} = 0.00015\% .$$

3.4 Backward Equations $T(p, s)$ and $v(p, s)$ for Subregions 3a and 3b

The Equations $T(p, s)$. The backward equation $T_{3a}(p, s)$ for subregion 3a has the following dimensionless form:

$$\frac{T_{3a}(p, s)}{T^*} = \theta_{3a}(\pi, \sigma) = \sum_{i=1}^{33} n_i (\pi + 0.240)^{I_i} (\sigma - 0.703)^{J_i} , \quad (6)$$

where $\theta = T/T^*$, $\pi = p/p^*$, and $\sigma = s/s^*$, with $T^* = 760$ K, $p^* = 100$ MPa, and $s^* = 4.4$ kJ kg⁻¹ K⁻¹. The coefficients n_i and exponents I_i and J_i of Eq. (6) are listed in Table 10.

The backward equation $T_{3b}(p, s)$ for subregion 3b reads in its dimensionless form

$$\frac{T_{3b}(p, s)}{T^*} = \theta_{3b}(\pi, \sigma) = \sum_{i=1}^{28} n_i (\pi + 0.760)^{I_i} (\sigma - 0.818)^{J_i} , \quad (7)$$

where $\theta = T/T^*$, $\pi = p/p^*$, and $\sigma = s/s^*$, with $T^* = 860$ K, $p^* = 100$ MPa, and $s^* = 5.3$ kJ kg⁻¹ K⁻¹. The coefficients n_i and exponents I_i and J_i of Eq. (7) are listed in Table 11.

Computer-program verification

To assist the user in computer-program verification of Eqs. (6) and (7), Table 12 contains test values for calculated temperatures.

Table 10. Coefficients and exponents of the backward equation $T_{3a}(p, s)$ for subregion 3a in its dimensionless form, Eq. (6)

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	-12	28	$0.150\ 042\ 008\ 263\ 875 \times 10^{10}$	18	-4	10	$-0.368\ 275\ 545\ 889\ 071 \times 10^3$
2	-12	32	$-0.159\ 397\ 258\ 480\ 424 \times 10^{12}$	19	-4	36	$0.664\ 768\ 904\ 779\ 177 \times 10^{16}$
3	-10	4	$0.502\ 181\ 140\ 217\ 975 \times 10^{-3}$	20	-2	1	$0.449\ 359\ 251\ 958\ 880 \times 10^{-1}$
4	-10	10	$-0.672\ 057\ 767\ 855\ 466 \times 10^2$	21	-2	4	$-0.422\ 897\ 836\ 099\ 655 \times 10^1$
5	-10	12	$0.145\ 058\ 545\ 404\ 456 \times 10^4$	22	-1	1	$-0.240\ 614\ 376\ 434\ 179$
6	-10	14	$-0.823\ 889\ 534\ 888\ 890 \times 10^4$	23	-1	6	$-0.474\ 341\ 365\ 254\ 924 \times 10^1$
7	-8	5	$-0.154\ 852\ 214\ 233\ 853$	24	0	0	$0.724\ 093\ 999\ 126\ 110$
8	-8	7	$0.112\ 305\ 046\ 746\ 695 \times 10^2$	25	0	1	$0.923\ 874\ 349\ 695\ 897$
9	-8	8	$-0.297\ 000\ 213\ 482\ 822 \times 10^2$	26	0	4	$0.399\ 043\ 655\ 281\ 015 \times 10^1$
10	-8	28	$0.438\ 565\ 132\ 635\ 495 \times 10^{11}$	27	1	0	$0.384\ 066\ 651\ 868\ 009 \times 10^{-1}$
11	-6	2	$0.137\ 837\ 838\ 635\ 464 \times 10^{-2}$	28	2	0	$-0.359\ 344\ 365\ 571\ 848 \times 10^{-2}$
12	-6	6	$-0.297\ 478\ 527\ 157\ 462 \times 10^1$	29	2	3	$-0.735\ 196\ 448\ 821\ 653$
13	-6	32	$0.971\ 777\ 947\ 349\ 413 \times 10^{13}$	30	3	2	$0.188\ 367\ 048\ 396\ 131$
14	-5	0	$-0.571\ 527\ 767\ 052\ 398 \times 10^{-4}$	31	8	0	$0.141\ 064\ 266\ 818\ 704 \times 10^{-3}$
15	-5	14	$0.288\ 307\ 949\ 778\ 420 \times 10^5$	32	8	1	$-0.257\ 418\ 501\ 496\ 337 \times 10^{-2}$
16	-5	32	$-0.744\ 428\ 289\ 262\ 703 \times 10^{14}$	33	10	2	$0.123\ 220\ 024\ 851\ 555 \times 10^{-2}$
17	-4	6	$0.128\ 017\ 324\ 848\ 921 \times 10^2$				

Table 11. Coefficients and exponents of the backward equation $T_{3b}(p, s)$ for subregion 3b in its dimensionless form, Eq. (7)

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	-12	1	$0.527\ 111\ 701\ 601\ 660$	15	-5	6	$0.880\ 531\ 517\ 490\ 555 \times 10^3$
2	-12	3	$-0.401\ 317\ 830\ 052\ 742 \times 10^2$	16	-4	12	$0.265\ 015\ 592\ 794\ 626 \times 10^7$
3	-12	4	$0.153\ 020\ 073\ 134\ 484 \times 10^3$	17	-3	1	$-0.359\ 287\ 150\ 025\ 783$
4	-12	7	$-0.224\ 799\ 398\ 218\ 827 \times 10^4$	18	-3	6	$-0.656\ 991\ 567\ 673\ 753 \times 10^3$
5	-8	0	$-0.193\ 993\ 484\ 669\ 048$	19	-2	2	$0.241\ 768\ 149\ 185\ 367 \times 10^1$
6	-8	1	$-0.140\ 467\ 557\ 893\ 768 \times 10^1$	20	0	0	$0.856\ 873\ 461\ 222\ 588$
7	-8	3	$0.426\ 799\ 878\ 114\ 024 \times 10^2$	21	2	1	$0.655\ 143\ 675\ 313\ 458$
8	-6	0	$0.752\ 810\ 643\ 416\ 743$	22	3	1	$-0.213\ 535\ 213\ 206\ 406$
9	-6	2	$0.226\ 657\ 238\ 616\ 417 \times 10^2$	23	4	0	$0.562\ 974\ 957\ 606\ 348 \times 10^{-2}$
10	-6	4	$-0.622\ 873\ 556\ 909\ 932 \times 10^3$	24	5	24	$-0.316\ 955\ 725\ 450\ 471 \times 10^{15}$
11	-5	0	$-0.660\ 823\ 667\ 935\ 396$	25	6	0	$-0.699\ 997\ 000\ 152\ 457 \times 10^{-3}$
12	-5	1	$0.841\ 267\ 087\ 271\ 658$	26	8	3	$0.119\ 845\ 803\ 210\ 767 \times 10^{-1}$
13	-5	2	$-0.253\ 717\ 501\ 764\ 397 \times 10^2$	27	12	1	$0.193\ 848\ 122\ 022\ 095 \times 10^{-4}$
14	-5	4	$0.485\ 708\ 963\ 532\ 948 \times 10^3$	28	14	2	$-0.215\ 095\ 749\ 182\ 309 \times 10^{-4}$

Table 12. Selected temperature values calculated from Eqs. (6) and (7)^a

Equation	p / MPa	s / kJ kg ⁻¹ K ⁻¹	T / K
$T_{3a}(p,s)$, Eq. (6)	20	3.8	$6.282\ 959\ 869 \times 10^2$
	50	3.6	$6.297\ 158\ 726 \times 10^2$
	100	4.0	$7.056\ 880\ 237 \times 10^2$
$T_{3b}(p,s)$, Eq. (7)	20	5.0	$6.401\ 176\ 443 \times 10^2$
	50	4.5	$7.163\ 687\ 517 \times 10^2$
	100	5.0	$8.474\ 332\ 825 \times 10^2$

^a It is recommended that programmed functions be verified using 8 byte real values for all variables.

The Equations $v(p,s)$. The backward equation $v_{3a}(p,s)$ for subregion 3a has the following dimensionless form:

$$\frac{v_{3a}(p,s)}{v^*} = \omega_{3a}(\pi, \sigma) = \sum_{i=1}^{28} n_i (\pi + 0.187)^{I_i} (\sigma - 0.755)^{J_i}, \quad (8)$$

where $\omega = v/v^*$, $\pi = p/p^*$, and $\sigma = s/s^*$, with $v^* = 0.0028 \text{ m}^3 \text{ kg}^{-1}$, $p^* = 100 \text{ MPa}$, and $s^* = 4.4 \text{ kJ kg}^{-1} \text{ K}^{-1}$. The coefficients n_i and exponents I_i and J_i of Eq. (8) are listed in Table 13.

The backward equation $v_{3b}(p,s)$ for subregion 3b reads in its dimensionless form

$$\frac{v_{3b}(p,s)}{v^*} = \omega_{3b}(\pi, \sigma) = \sum_{i=1}^{31} n_i (\pi + 0.298)^{I_i} (\sigma - 0.816)^{J_i}, \quad (9)$$

where $\omega = v/v^*$, $\pi = p/p^*$, and $\sigma = s/s^*$, with $v^* = 0.0088 \text{ m}^3 \text{ kg}^{-1}$, $p^* = 100 \text{ MPa}$, and $s^* = 5.3 \text{ kJ kg}^{-1} \text{ K}^{-1}$. The coefficients n_i and exponents I_i and J_i of Eq. (9) are listed in Table 14.

Computer-program verification. To assist the user in computer-program verification of Eqs. (8) and (9), Table 15 contains test values for calculated specific volumes.

Table 13. Coefficients and exponents of the backward equation $v_{3a}(p, s)$ for subregion 3a in its dimensionless form, Eq. (8).

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	-12	10	0.795 544 074 093 975 $\times 10^2$	15	-3	2	-0.118 008 384 666 987
2	-12	12	-0.238 261 242 984 590 $\times 10^4$	16	-3	4	0.253 798 642 355 900 $\times 10^1$
3	-12	14	0.176 813 100 617 787 $\times 10^5$	17	-2	3	0.965 127 704 669 424
4	-10	4	-0.110 524 727 080 379 $\times 10^{-2}$	18	-2	8	-0.282 172 420 532 826 $\times 10^2$
5	-10	8	-0.153 213 833 655 326 $\times 10^2$	19	-1	1	0.203 224 612 353 823
6	-10	10	0.297 544 599 376 982 $\times 10^3$	20	-1	2	0.110 648 186 063 513 $\times 10^1$
7	-10	20	-0.350 315 206 871 242 $\times 10^8$	21	0	0	0.526 127 948 451 280
8	-8	5	0.277 513 761 062 119	22	0	1	0.277 000 018 736 321
9	-8	6	-0.523 964 271 036 888	23	0	3	0.108 153 340 501 132 $\times 10^1$
10	-8	14	-0.148 011 182 995 403 $\times 10^6$	24	1	0	-0.744 127 885 357 893 $\times 10^{-1}$
11	-8	16	0.160 014 899 374 266 $\times 10^7$	25	2	0	0.164 094 443 541 384 $\times 10^{-1}$
12	-6	28	0.170 802 322 663 427 $\times 10^{13}$	26	4	2	-0.680 468 275 301 065 $\times 10^{-1}$
13	-5	1	0.246 866 996 006 494 $\times 10^{-3}$	27	5	2	0.257 988 576 101 640 $\times 10^{-1}$
14	-4	5	0.165 326 084 797 980 $\times 10^1$	28	6	0	-0.145 749 861 944 416 $\times 10^{-3}$

Table 14. Coefficients and exponents of the backward equation $v_{3b}(p, s)$ for subregion 3b in its dimensionless form, Eq. (9)

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	-12	0	0.591 599 780 322 238 $\times 10^{-4}$	17	-4	2	-0.121 613 320 606 788 $\times 10^2$
2	-12	1	-0.185 465 997 137 856 $\times 10^{-2}$	18	-4	3	0.167 637 540 957 944 $\times 10^1$
3	-12	2	0.104 190 510 480 013 $\times 10^{-1}$	19	-3	1	-0.744 135 838 773 463 $\times 10^1$
4	-12	3	0.598 647 302 038 590 $\times 10^{-2}$	20	-2	0	0.378 168 091 437 659 $\times 10^{-1}$
5	-12	5	-0.771 391 189 901 699	21	-2	1	0.401 432 203 027 688 $\times 10^1$
6	-12	6	0.172 549 765 557 036 $\times 10^1$	22	-2	2	0.160 279 837 479 185 $\times 10^2$
7	-10	0	-0.467 076 079 846 526 $\times 10^{-3}$	23	-2	3	0.317 848 779 347 728 $\times 10^1$
8	-10	1	0.134 533 823 384 439 $\times 10^{-1}$	24	-2	4	-0.358 362 310 304 853 $\times 10^1$
9	-10	2	-0.808 094 336 805 495 $\times 10^{-1}$	25	-2	12	-0.115 995 260 446 827 $\times 10^7$
10	-10	4	0.508 139 374 365 767	26	0	0	0.199 256 573 577 909
11	-8	0	0.128 584 643 361 683 $\times 10^{-2}$	27	0	1	-0.122 270 624 794 624
12	-5	1	-0.163 899 353 915 435 $\times 10^1$	28	0	2	-0.191 449 143 716 586 $\times 10^2$
13	-5	2	0.586 938 199 318 063 $\times 10^1$	29	1	0	-0.150 448 002 905 284 $\times 10^{-1}$
14	-5	3	-0.292 466 667 918 613 $\times 10^1$	30	1	2	0.146 407 900 162 154 $\times 10^2$
15	-4	0	-0.614 076 301 499 537 $\times 10^{-2}$	31	2	2	-0.327 477 787 188 230 $\times 10^1$
16	-4	1	0.576 199 014 049 172 $\times 10^1$				

Table 15. Selected specific volume values calculated from Eqs. (8) and (9) ^a

Equation	p / MPa	$s / \text{kJ kg}^{-1} \text{K}^{-1}$	$v / \text{m}^3 \text{kg}^{-1}$
$v_{3a}(p, s)$, Eq. (8)	20	3.8	$1.733\,791\,463 \times 10^{-3}$
	50	3.6	$1.469\,680\,170 \times 10^{-3}$
	100	4.0	$1.555\,893\,131 \times 10^{-3}$
$v_{3b}(p, s)$, Eq. (9)	20	5.0	$6.262\,101\,987 \times 10^{-3}$
	50	4.5	$2.332\,634\,294 \times 10^{-3}$
	100	5.0	$2.449\,610\,757 \times 10^{-3}$

^a It is recommended that programmed functions be verified using 8 byte real values for all variables.

Numerical Consistency with the Basic Equation of IAPWS-IF97. The maximum temperature differences and related root-mean-square differences between the temperatures calculated from Eqs. (6) and (7) and the IAPWS-IF97 basic equation $f_3^{97}(v, T)$ in comparison with the permissible differences are listed in Table 16.

Table 16 also contains the maximum relative deviations and root-mean-square relative deviations for the specific volume of Eqs. (8) and (9) from IAPWS-IF97.

The critical temperature and the critical volume are met exactly by the equations $T(p, s)$ and $v(p, s)$.

Table 16. Maximum differences and root-mean-square differences of the temperature calculated from Eqs. (6) and (7), and specific volume calculated from Eqs. (8) and (9) from the IAPWS-IF97 basic equation $f_3^{97}(v, T)$, and related permissible values

Subregion	Equation	$ \Delta T _{\text{tol}}$	$ \Delta T _{\text{max}}$	$ \Delta T _{\text{RMS}}$
3a	(6)	25 mK	24.8 mK	11.2 mK
3b	(7)	25 mK	22.1 mK	10.1 mK
Subregion	Equation	$ \Delta v/v _{\text{tol}}$	$ \Delta v/v _{\text{max}}$	$ \Delta v/v _{\text{RMS}}$
3a	(8)	0.01 %	0.0096 %	0.0052 %
3b	(9)	0.01 %	0.0077 %	0.0037 %

Consistency at Boundary Between Subregions. The maximum temperature difference between the two backward equations, Eq. (6) and Eq. (7), along the boundary s_c , has the following value

$$|\Delta T|_{\text{max}} = |T_{3a}(p, s_c) - T_{3b}(p, s_c)|_{\text{max}} = 0.093 \text{ mK}.$$

Thus, the temperature differences between the two backward functions $T(p, s)$ of the adjacent subregions are smaller than their differences with the IAPWS-IF97 equations.

The relative specific volume differences between the two backward equations $v(p, s)$, Eqs. (8) and (9), of the adjacent subregions are also smaller than the differences of these equations with the IAPWS-IF97 basic equation. Along the boundary s_c , the maximum difference between the corresponding equations was determined as

$$\left| \frac{\Delta v}{v} \right|_{\max} = \left| \frac{v_{3a}(p, s_c) - v_{3b}(p, s_c)}{v_{3b}(p, s_c)} \right|_{\max} = 0.00046\% .$$

3.5 Computing Time in Relation to IAPWS-IF97

A very important motivation for the development of the backward equations $T(p, h)$, $v(p, h)$ and $T(p, s)$, $v(p, s)$ for region 3 was reducing the computing time to obtain thermodynamic properties and differential quotients from given variables (p, h) and (p, s) . In IAPWS-IF97, time-consuming iterations, *e.g.*, the two-dimensional Newton method, are required. Using the $T_3(p, h)$, $v_3(p, h)$, $T_3(p, s)$ and $v_3(p, s)$ equations, the calculation speed is about 20 times faster than that of the two-dimensional Newton method with convergence tolerances set to the values shown in Table 1.

The numerical consistency of T and v obtained in this way is sufficient for most heat cycle calculations.

For users not satisfied with the numerical consistency of the backward equations, the equations are still recommended for generating starting points for the iterative process. They will significantly reduce the time required to reach the convergence criteria of the iteration.

4 Boundary Equations $p_{\text{sat}}(h)$ and $p_{\text{sat}}(s)$ for the Saturation Lines of Region 3

4.1 Determination of the Region Boundaries for Given Variables (p, h) and (p, s)

The boundaries between region 3 and the two-phase region 4 are the saturated liquid line $x = 0$ and saturated vapor line $x = 1$; see Figures 3 and 4. A one-dimensional iteration using the IAPWS-IF97 basic equation $f_3^{97}(v, T)$ and the saturation-pressure equation $p_{\text{sat}}^{97}(T)$ is required to calculate the enthalpy or entropy from a given pressure on the saturated liquid or saturated vapor lines of region 3. The boundary equations $p_{3\text{sat}}(h)$ and $p_{3\text{sat}}(s)$, provided in this release, make it possible to determine without iteration whether the given state point is located in the two-phase region 4 or in the single-phase region 3.

The boundary between regions 1 and 3 can be calculated directly from a given pressure p and from $T = 623.15$ K using the IAPWS-IF97 basic equation $g_1^{97}(p, T)$. The boundary between regions 2 and 3 can be calculated directly from given pressure p and from the B23-equation $T = T_{\text{B23}}^{97}(p)$ of IAPWS-IF97 and using the IAPWS-IF97 basic equation $g_2^{97}(p, T)$.

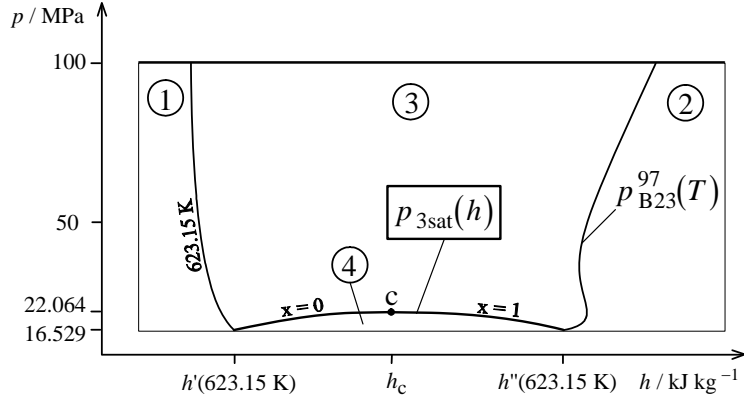


Figure 3. Illustration of IAPWS-IF97 region 3 and the boundary equation $p_{3\text{sat}}(h)$ in a p - h diagram

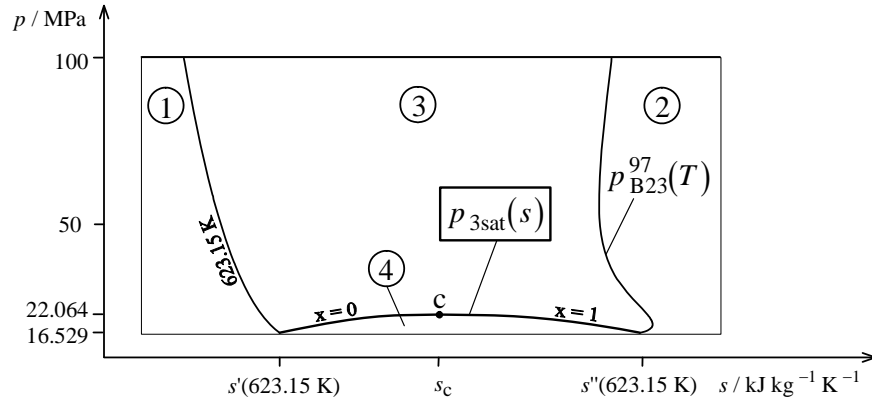


Figure 4. Illustration of IAPWS-IF97 region 3 and the boundary equation $p_{3\text{sat}}(s)$ in a p - s diagram

4.2 Numerical Consistency Requirements

The required consistency of the boundary equations for the saturation lines of region 3 result from IAPWS requirements on backward functions. Therefore, the backward functions $T(p, h)$, $v(p, h)$, $T(p, s)$, and $v(p, s)$ have to fulfill their numerical consistency requirements when using the boundary equations $p_{3\text{sat}}(h)$ and $p_{3\text{sat}}(s)$ for determining the region of a given state point.

4.3 Boundary Equations $p_{\text{sat}}(h)$ and $p_{\text{sat}}(s)$

The Equation $p_{3\text{sat}}(h)$. The equation $p_{3\text{sat}}(h)$ describes the saturated liquid line and the saturated vapor line including the critical point in the enthalpy range (see Figure 3):

$$h'(623.15 \text{ K}) \leq h \leq h''(623.15 \text{ K}),$$

$$\text{where } h'(623.15 \text{ K}) = 1.670\,858\,218 \times 10^3 \text{ kJ kg}^{-1},$$

$$\text{and } h''(623.15 \text{ K}) = 2.563\,592\,004 \times 10^3 \text{ kJ kg}^{-1}.$$

The boundary equation $p_{3\text{sat}}(h)$ has the following dimensionless form:

$$\frac{p_{3\text{sat}}(h)}{p^*} = \pi(\eta) = \sum_{i=1}^{14} n_i (\eta - 1.02)^{I_i} (\eta - 0.608)^{J_i}, \quad (10)$$

where $\pi = p/p^*$ and $\eta = h/h^*$, with $p^* = 22 \text{ MPa}$ and $h^* = 2600 \text{ kJ kg}^{-1}$. The coefficients n_i and exponents I_i and J_i of Eq. (10) are listed in Table 17.

Table 17. Coefficients and exponents of the boundary equation $p_{3\text{sat}}(h)$ in its dimensionless form, Eq. (10)

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	0	0	0.600 073 641 753 024	8	8	24	0.252 304 969 384 128 $\times 10^{18}$
2	1	1	-0.936 203 654 849 857 $\times 10^1$	9	14	16	-0.389 718 771 997 719 $\times 10^{19}$
3	1	3	0.246 590 798 594 147 $\times 10^2$	10	20	16	-0.333 775 713 645 296 $\times 10^{23}$
4	1	4	-0.107 014 222 858 224 $\times 10^3$	11	22	3	0.356 499 469 636 328 $\times 10^{11}$
5	1	36	-0.915 821 315 805 768 $\times 10^{14}$	12	24	18	-0.148 547 544 720 641 $\times 10^{27}$
6	5	3	-0.862 332 011 700 662 $\times 10^4$	13	28	8	0.330 611 514 838 798 $\times 10^{19}$
7	7	0	-0.235 837 344 740 032 $\times 10^2$	14	36	24	0.813 641 294 467 829 $\times 10^{38}$

Computer-program verification. To assist the user in computer-program verification of Eq. (10), Table 18 contains test values for calculated pressures.

Table 18. Selected pressure values calculated from Eq. (10)^a

Equation	$h / \text{kJ kg}^{-1}$	p / MPa
$p_{3\text{sat}}(h)$, Eq. (10)	1700	1.724 175 718 $\times 10^1$
	2000	2.193 442 957 $\times 10^1$
	2400	2.018 090 839 $\times 10^1$

^a It is recommended that programmed functions be verified using 8 byte real values for all variables.

Equation $p_{3\text{sat}}(s)$. The equation $p_{3\text{sat}}(s)$ describes the saturated liquid line and the saturated vapor line including the critical point in the entropy range (see Figure 4):

$$s'(623.15 \text{ K}) \leq s \leq s''(623.15 \text{ K}),$$

$$\text{where } s'(623.15 \text{ K}) = 3.778\,281\,340 \text{ kJ kg}^{-1} \text{ K}^{-1},$$

$$\text{and } s''(623.15 \text{ K}) = 5.210\,887\,825 \text{ kJ kg}^{-1} \text{ K}^{-1}.$$

The boundary equation $p_{3\text{sat}}(s)$ has the following dimensionless form:

$$\frac{p_{3\text{sat}}(s)}{p^*} = \pi(\sigma) = \sum_{i=1}^{10} n_i (\sigma - 1.03)^{I_i} (\sigma - 0.699)^{J_i}, \quad (11)$$

where $\pi = p/p^*$ and $\sigma = s/s^*$, with $p^* = 22 \text{ MPa}$ and $s^* = 5.2 \text{ kJ kg}^{-1} \text{ K}^{-1}$. The coefficients n_i and exponents I_i and J_i of Eq. (11) are listed in Table 19.

Table 19. Coefficients and exponents of the boundary equation $p_{3\text{sat}}(s)$ in its dimensionless form, Eq. (11)

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	0	0	0.639 767 553 612 785	6	12	14	$-0.378 829 107 169 011 \times 10^{18}$
2	1	1	$-0.129 727 445 396 014 \times 10^2$	7	16	36	$-0.955 586 736 431 328 \times 10^{35}$
3	1	32	$-0.224 595 125 848 403 \times 10^{16}$	8	24	10	$0.187 269 814 676 188 \times 10^{24}$
4	4	7	$0.177 466 741 801 846 \times 10^7$	9	28	0	$0.119 254 746 466 473 \times 10^{12}$
5	12	4	$0.717 079 349 571 538 \times 10^{10}$	10	32	18	$0.110 649 277 244 882 \times 10^{37}$

Computer-program verification. To assist the user in computer-program verification of Eq. (11), Table 20 contains test values for calculated pressures.

Table 20. Selected pressure values calculated from Eq. (11)^a

Equation	$s / \text{kJ kg}^{-1} \text{K}^{-1}$	p / MPa
$p_{3\text{sat}}(s)$, Eq. (11)	3.8	$1.687 755 057 \times 10^1$
	4.2	$2.164 451 789 \times 10^1$
	5.2	$1.668 968 482 \times 10^1$

^a It is recommended that programmed functions be verified using 8 byte real values for all variables.

Numerical Consistency with the Saturation-Pressure Equation of IAPWS-IF97. The maximum percentage deviation between the pressure calculated from the boundary equation $p_{3\text{sat}}(h)$, Eq. (10), and the IAPWS-IF97 saturation-pressure equation $p_{\text{sat}}^{97}(T)$ has the following value

$$\left| \frac{\Delta p}{p} \right|_{\max} = \left| \frac{p_{3\text{sat}}(h) - p_{\text{sat}}^{97}(T)}{p_{\text{sat}}^{97}(T)} \right|_{\max} = 0.00043 \%$$

The maximum percentage deviation between the calculated pressure $p_{3\text{sat}}(s)$, Eq. (11), and the IAPWS-IF97 saturation-pressure equation $p_{\text{sat}}^{97}(T)$ has the following value

$$\left| \frac{\Delta p}{p} \right|_{\text{max}} = \left| \frac{p_{3\text{sat}}(s) - p_{\text{sat}}^{97}(T)}{p_{\text{sat}}^{97}(T)} \right|_{\text{max}} = 0.0033 \% .$$

Consistency of the Backward Equations $T(p,h)$, $v(p,h)$, $T(p,s)$, and $v(p,s)$ with the Basic Equation of IAPWS-IF97 along the Boundary Equations $p_{\text{sat}}(h)$ and $p_{\text{sat}}(s)$. The maximum temperature differences between the backward equations $T_{3a}(p,h)$, Eq. (2), and $T_{3b}(p,h)$, Eq. (3), and the IAPWS-IF97 basic equation $f_3^{97}(v,T)$ along the boundary equation $p_{3\text{sat}}(h)$, Eq. (10), in comparison with the permissible differences are listed in Table 21. The temperature differences were calculated as $\Delta T = T_3 \left[p_{3\text{sat}}(h_3^{97}), h_3^{97} \right] - T$. The function T_3 represents the calculation of $T(p,h)$ using the backward equations of subregions 3a and 3b, Eqs. (2) and (3).

Table 21. Maximum differences of temperature and specific volume calculated from Eqs. (2), (3), (4), and (5) from the IAPWS-IF97 basic equation $f_3^{97}(v,T)$ along the boundary equation $p_{3\text{sat}}(h)$, Eq. (10), and related permissible values

Subregion	Equation	$ \Delta T _{\text{tol}}$	$ \Delta T _{\text{max}}$
3a	(2)	25 mK	0.47 mK
3b	(3)	25 mK	0.46 mK
Subregion	Equation	$ \Delta v/v _{\text{tol}}$	$ \Delta v/v _{\text{max}}$
3a	(4)	0.01 %	0.00077 %
3b	(5)	0.01 %	0.0012 %

Furthermore, Table 21 contains the maximum percentage differences of specific volume between the backward equations $v_{3a}(p,h)$, Eq. (4), and $v_{3b}(p,h)$, Eq. (5), and the IAPWS-IF97 basic equation $f_3^{97}(v,T)$ along the boundary equation $p_{3\text{sat}}(h)$, Eq. (10). The relative differences of specific volume were calculated as $\Delta v/v = \left(v_3 \left[p_{3\text{sat}}(h_3^{97}), h_3^{97} \right] - v \right) / v$. The function v_3 represents the calculation of $v(p,h)$ using the backward equations of subregions 3a and 3b, Eqs. (4) and (5).

The maximum temperature differences and the maximum relative differences of specific volume are smaller than the permissible values. Therefore, the numerical consistency of the boundary equation $p_{3\text{sat}}(h)$, Eq. (10), is sufficient.

The maximum temperature differences between the backward equations $T_{3a}(p, s)$, Eq. (6), and $T_{3b}(p, s)$, Eq. (7), and the IAPWS-IF97 basic equation $f_3^{97}(v, T)$ along the boundary equation $p_{3\text{sat}}(s)$, Eq. (11), in comparison with the permissible differences are listed in Table 22. The temperature differences were calculated as $\Delta T = T_3 \left[p_{3\text{sat}}(s_3^{97}), s_3^{97} \right] - T$. The function T_3 represents the calculation of $T(p, s)$ using the backward equations of subregions 3a and 3b, Eqs. (6) and (7).

Table 22. Maximum differences of temperature and specific volume calculated from Eqs. (6), (7), (8), and (9) to the IAPWS-IF97 basic equation $f_3^{97}(v, T)$ along the boundary equation $p_{3\text{sat}}(s)$, Eq. (11), and related permissible values

Subregion	Equation	$ \Delta T _{\text{tol}}$	$ \Delta T _{\text{max}}$
3a	(6)	25 mK	2.69 mK
3b	(7)	25 mK	2.12 mK
Subregion	Equation	$ \Delta v/v _{\text{tol}}$	$ \Delta v/v _{\text{max}}$
3a	(8)	0.01 %	0.0034 %
3b	(9)	0.01 %	0.0020 %

Furthermore, Table 22 contains the maximum percentage differences of specific volume between the backward equations $v_{3a}(p, s)$, Eq. (8), and $v_{3b}(p, s)$, Eq. (9), and the IAPWS-IF97 basic equation $f_3^{97}(v, T)$ along the boundary equation $p_{3\text{sat}}(s)$, Eq. (11). The relative differences of specific volume were calculated as $\Delta v/v = \left(v_3 \left[p_{3\text{sat}}(s_3^{97}), s_3^{97} \right] - v \right) / v$. The function v_3 represents the calculation of $v(p, s)$ using the backward equations of subregions 3a and 3b, Eqs. (8) and (9).

The maximum temperature differences and the maximum relative differences of specific volume are smaller than the permissible values. Therefore, the numerical consistency of the boundary equation $p_{3\text{sat}}(s)$, Eq. (11), is sufficient.

4.4 Computing Time in Relation to IAPWS-IF97

A very important motivation for the development of the equations for saturation lines of region 3 was reducing the computing time to determine the region for a given state point (p, h) and (p, s) . In IAPWS-IF97, time-consuming iterations, *e.g.*, the Newton method, are required. By using equations $p_{3\text{sat}}(h)$, Eq. (10), and $p_{3\text{sat}}(s)$, Eq. (11), the calculation to determine the region is about 60 times faster than that of the two-dimensional Newton method.

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