

The IAPWS Guideline on the Fast Calculation of Steam and Water Properties with the Spline-Based Table Look-up Method (SBTL)

Project of the IAPWS Task Group “CFD Steam Property Formulation”

Hans-Joachim Kretzschmar, Matthias Kunick, Zittau/Goerlitz University of Applied Sciences

Jan Hrubý (Chair), Michal Duška, Václav Vinš, Czech Academy of Sciences, Prague

Francesca di Mare, German Aerospace Center (DLR), Cologne

Anurag Singh, General Electric, Schenectady

Evaluation Committee:

Adam Novy, Doosan Skoda (Chair)

Kiyoshi Miyagawa

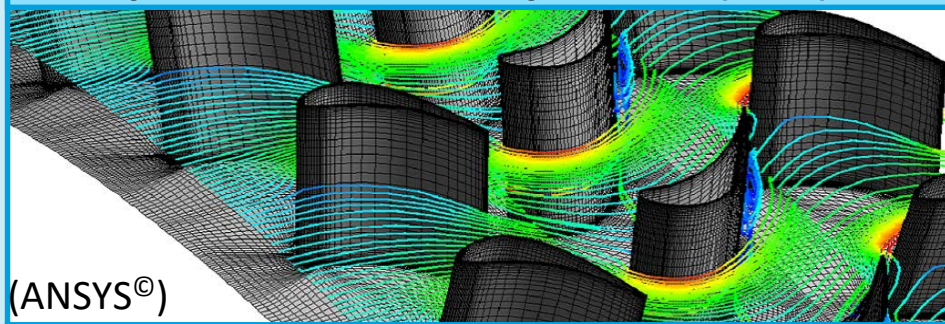
Reiner Pawellek, STEAG

Francisco Blangetti, Alstom Power

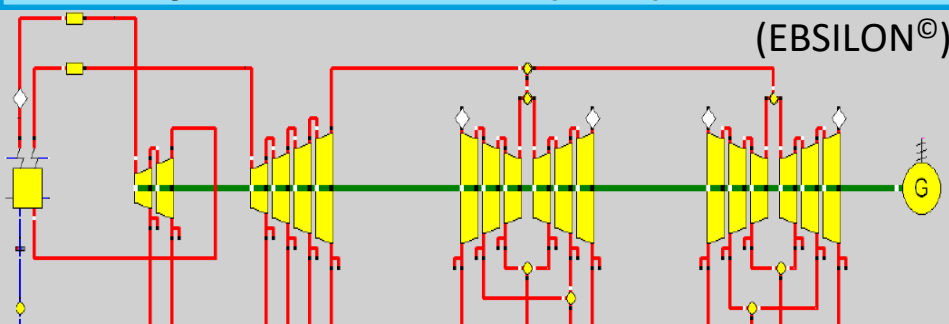
Julien Bonifay & Ingo Weber, Siemens Energy

Requirements for Property Calculation Algorithms in Extensive Process Simulations

Computational Fluid Dynamics (CFD)



Heat Cycle Calculations (HCC)



Accuracy:

Deviations in calculated fluid properties lead to inaccurate mass, energy, and entropy balances.

⇒ Property functions need to be very accurate!

Computing times:

Property functions are called extremely often, which consumes a large share of the overall computing time.

⇒ Property functions need to be very fast!

Numerical Consistency / Differentiability:

Numerical solvers require continuity and numerical consistency of the equations to be solved.

⇒ Property functions need to be numerically consistent and continuous differentiable!

Available IAPWS-Formulations for Water and Steam – Multi-parameter Equations of State

Computing times:

Computing Time Ratio:
$$CTR = \frac{\text{Computing time of calculation from Peng-Robinson EOS}}{\text{Computing time of calculation from considered EOS}}$$

ANI = Average number of iterations

$p(v,u)$				
EOS	IF97 region 1 (liquid)		IF97 region 2 (vapor)	
	ANI	CTR	ANI	CTR
Ideal gas	-	-	3.83	1.47
PR - EOS	3	1.00	3.08	1.00
IAPWS-IF97	2.94	1/6.08	3.64	1/11.1
IAPWS-95	2.94	1/89.6	3.68	1/107

(Assumption: phase is known)

⇒ **Multi-parameter equations of state are too slow for CFD simulations!**

Convergence criteria:

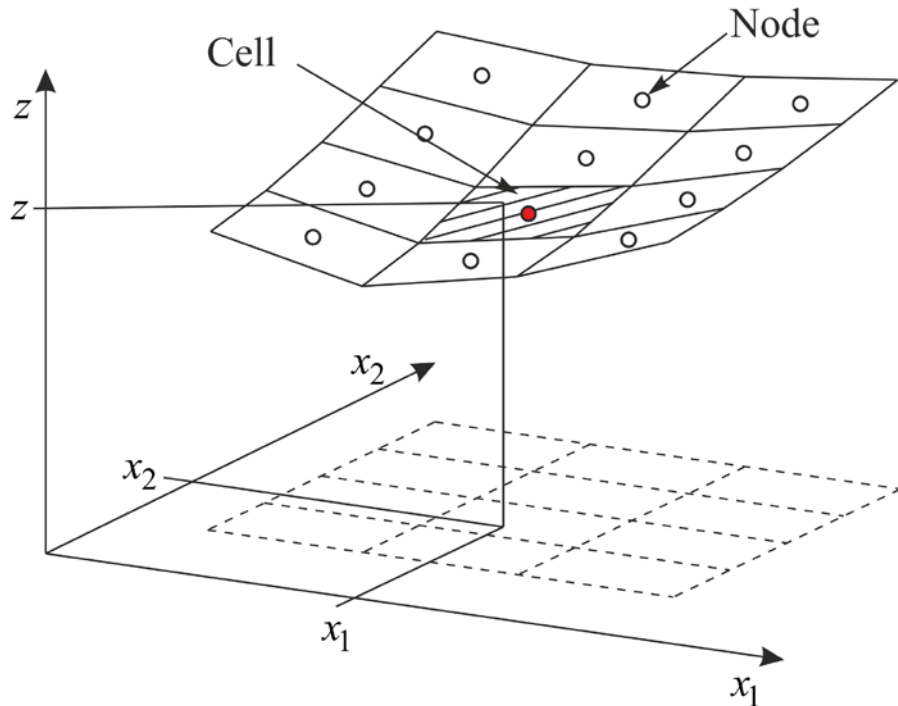
$$\left| \frac{\Delta v}{v} \right| \leq 10^{-6}$$

$$u \leq 1 \text{ kJ/kg: } |\Delta u| \leq 10^{-6} \text{ kJ/kg}$$

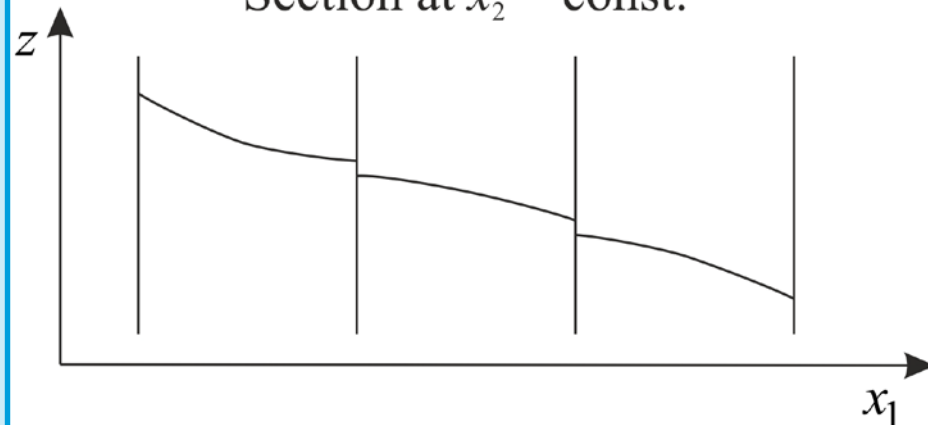
$$u > 1 \text{ kJ/kg: } \left| \frac{\Delta u}{u} \right| \leq 10^{-6}$$

Available IAPWS-Formulations for Water and Steam – Table Look-up Methods

Interpolation of any property function $z(x_1, x_2)$:



Section at $x_2 = \text{const.}$



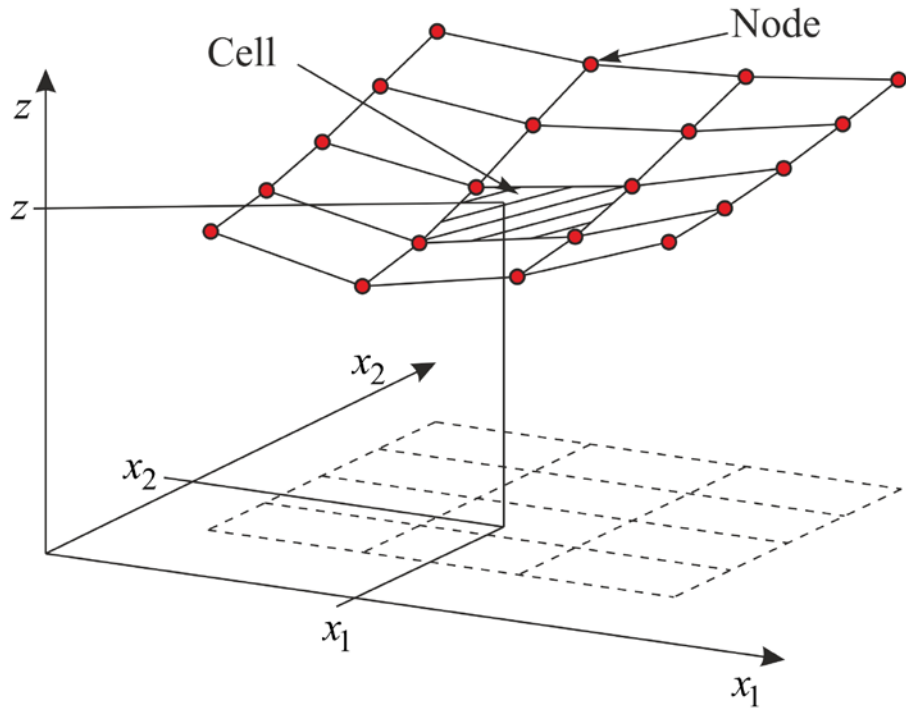
- Tabulation of the desired fluid properties in a grid of nodes, where the values are calculated from a reference eq. of state.
- Calculation during process simulation:
 - Cell-search in the grid of nodes
 - Interpolation of $z(x_1, x_2)$:
 - **local methods**, for example:
 - bi-linear (C^0 -continuous), z.B. ANSYS-CFX
 - **TTSE (Taylor series expansion - discontinuous)**

$$z_{\{i,j\}}(x_1, x_2) = \sum_{k=1}^2 \sum_{l=1}^2 a_{ijkl} (x_1 - x_{1,i})^{k-1} (x_2 - x_{2,j})^{l-1}$$

$$z_{\{i,j\}}(x_1, x_2) = z_{i,j} + \Delta x_{1,i} \left(\frac{\partial z}{\partial x_1} \right)_{x_2} + \frac{\Delta x_{1,i}^2}{2} \left(\frac{\partial^2 z}{\partial x_1^2} \right)_{x_2} \\ + \Delta x_{2,j} \left(\frac{\partial z}{\partial x_2} \right)_{x_1} + \frac{\Delta x_{2,j}^2}{2} \left(\frac{\partial^2 z}{\partial x_2^2} \right)_{x_1} + \Delta x_{1,i} \Delta x_{2,j} \left(\frac{\partial^2 z}{\partial x_1 \partial x_2} \right)$$

Available IAPWS-Formulations for Water and Steam – Table Look-up Methods

Interpolation of any property function $z(x_1, x_2)$:



- Tabulation of the desired fluid properties in a grid of nodes, where the values are calculated from a reference eq. of state.
- Calculation during process simulation:
 - Cell-search in the grid of nodes
 - Interpolation of $z(x_1, x_2)$:
 - local methods, for example:
 - bi-linear (C^0 -continuous), z.B. ANSYS-CFX
 - TTSE (Taylor series expansion - discontinuous)
 - **global methods**, for example:
bi-cubic spline function (C^2 -continuous).

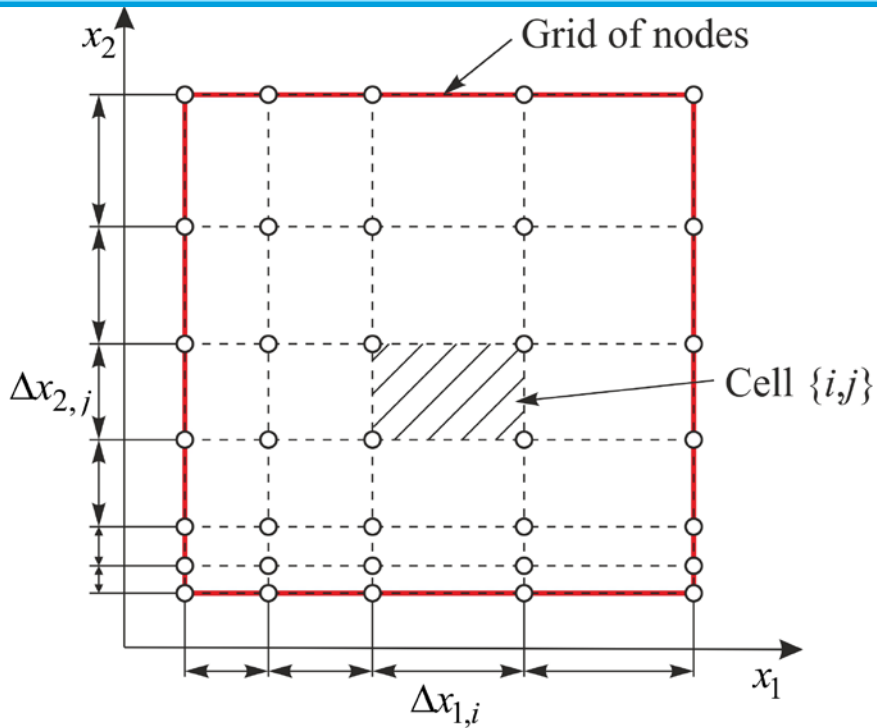
⇒ Accuracy can be controlled with the number of nodes in the grid.

⇒ Computing times are mostly determined by the cell-search algorithm.

⇒ Look-up tables can easily be established using an existing equations of state.
→ Table look-up methods are very flexible!

Available IAPWS-Formulations for Water and Steam – Table Look-up Methods

Interpolation of any property function $z(x_1, x_2)$:



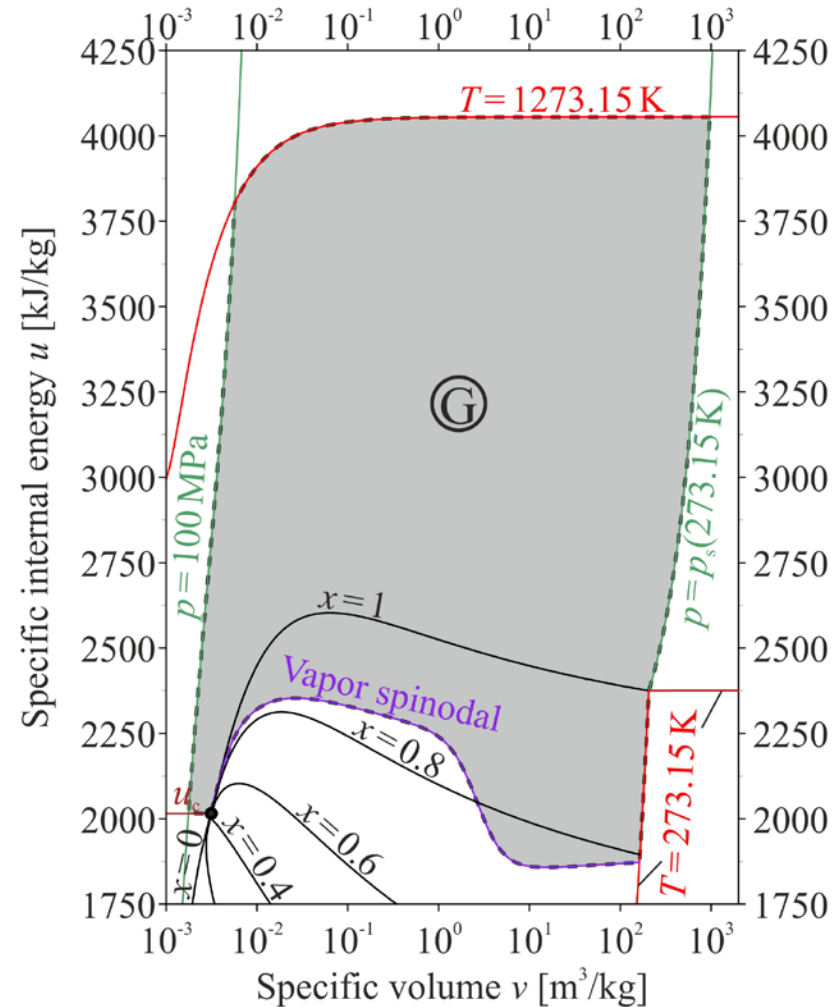
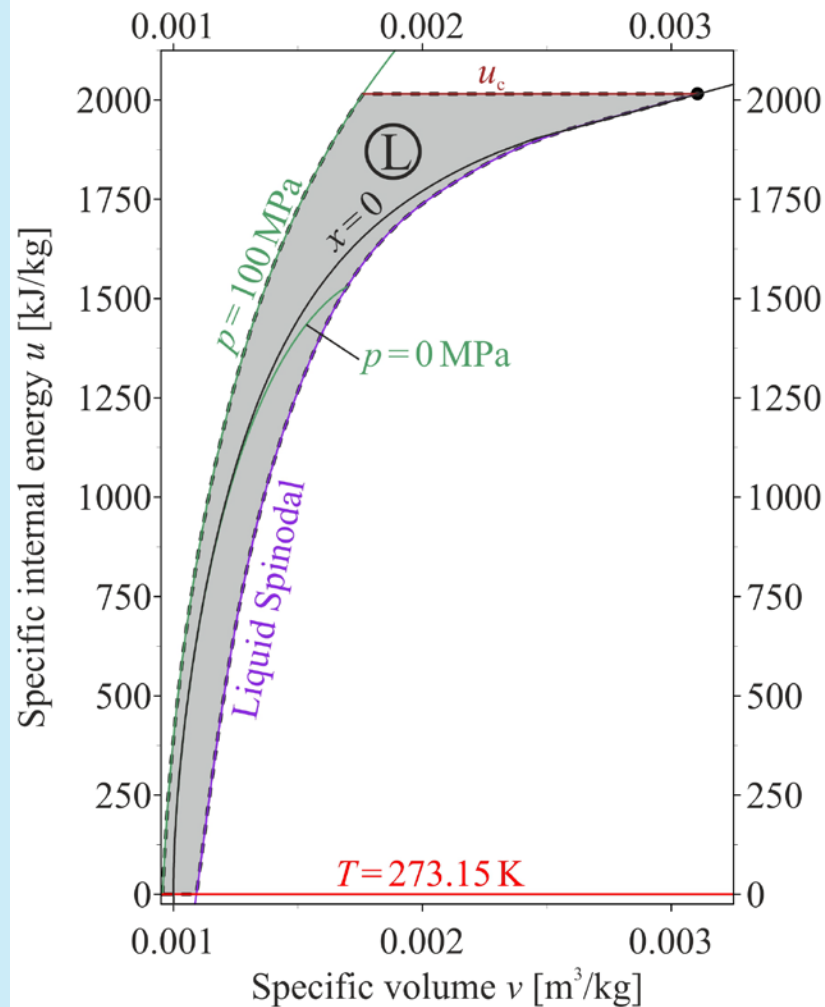
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 - local methods, for example:
 - bi-linear (C^0 -continuous), z.B. ANSYS-CFX
 - TTSE (Taylor series expansion - discontinuous)
 - global methods, for example:
bi-cubic spline function (C^2 -continuous).

Frequent problems:

- To take the non-linearity of the property function into account, the node density varies across the grid (non-equidistant nodes) → extensive cell-search (computing times ↑)
- Polynomials of third or higher order → computationally intensive inverse functions
- Global methods require a grid of nodes with a rectangular outer boundary → unfavorable approaches to describe non-rectangular domains are often in use

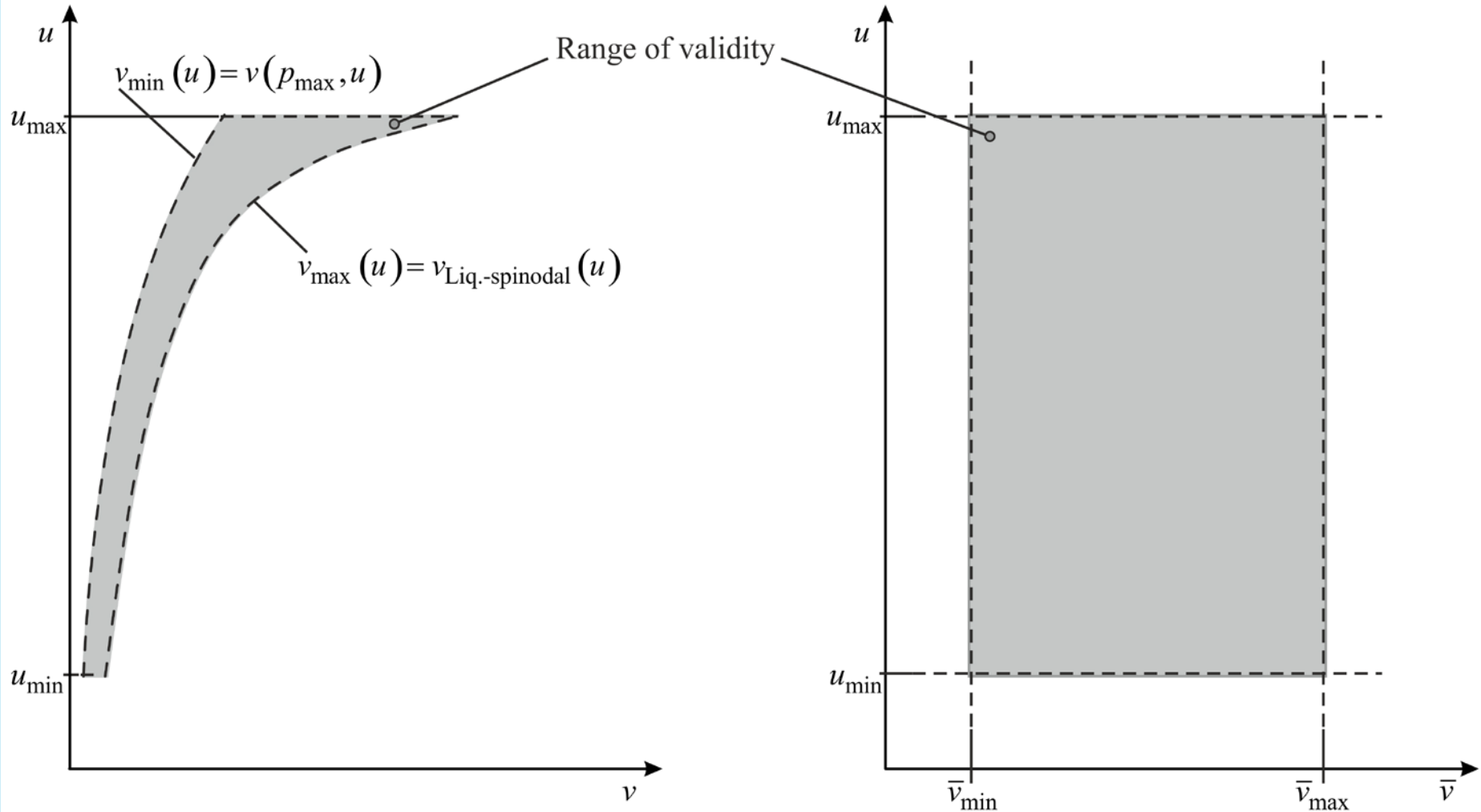
The Spline-Based Table Look-up Method (SBTL)

Generation of a Spline-Function $p^{\text{SPL}}(v, u)$ based on the IAPWS-95:



The Spline-Based Table Look-up Method (SBTL)

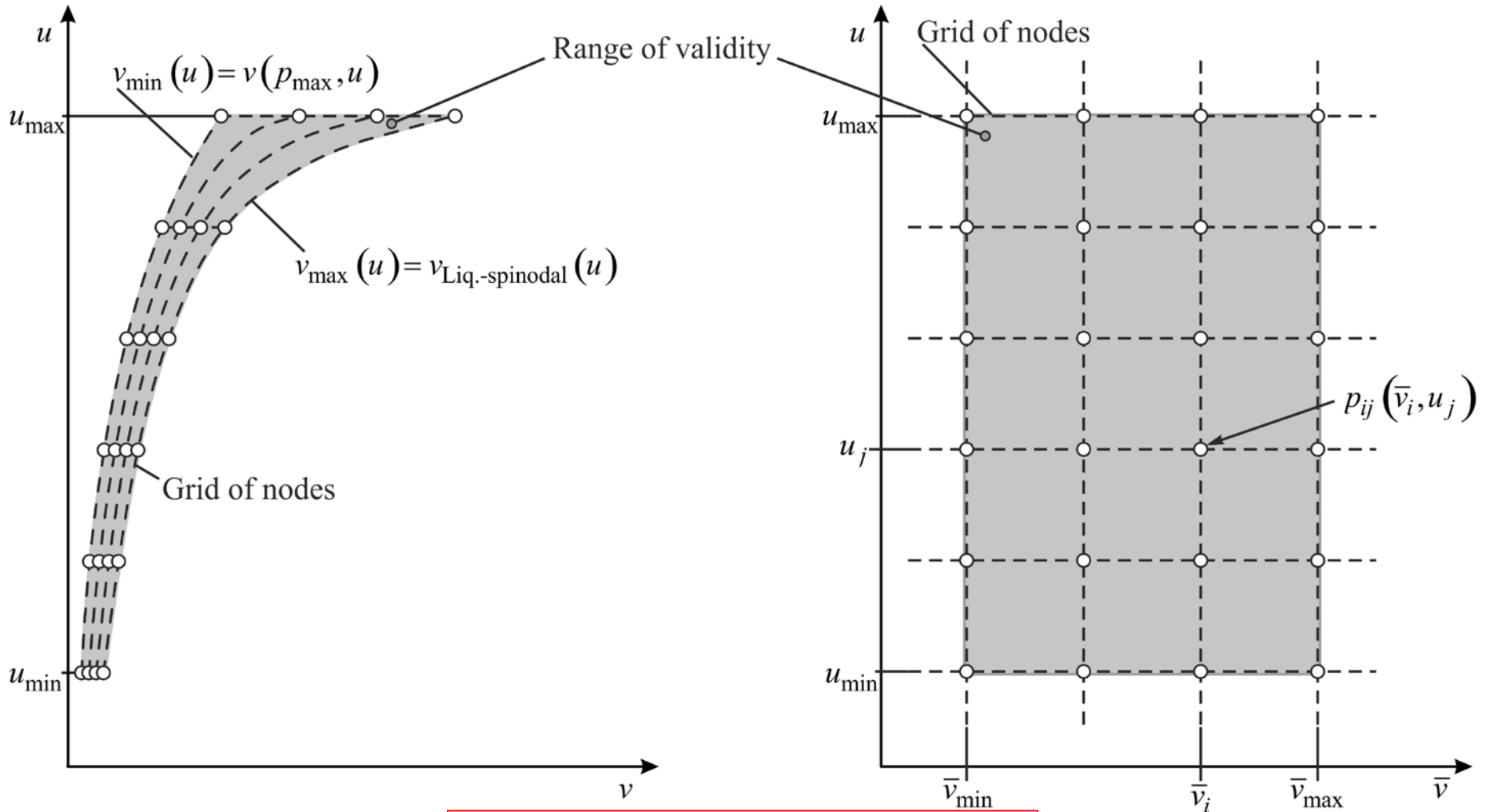
Generation of a Spline-Function $p^{\text{SPL}}(v, u)$ based on the IAPWS-95:



$$\bar{v}(v, u) = \frac{\bar{v}_{\max} - \bar{v}_{\min}}{v_{\max}(u) - v_{\min}(u)} (v - v_{\min}(u)) + \bar{v}_{\min}$$

The Spline-Based Table Look-up Method (SBTL)

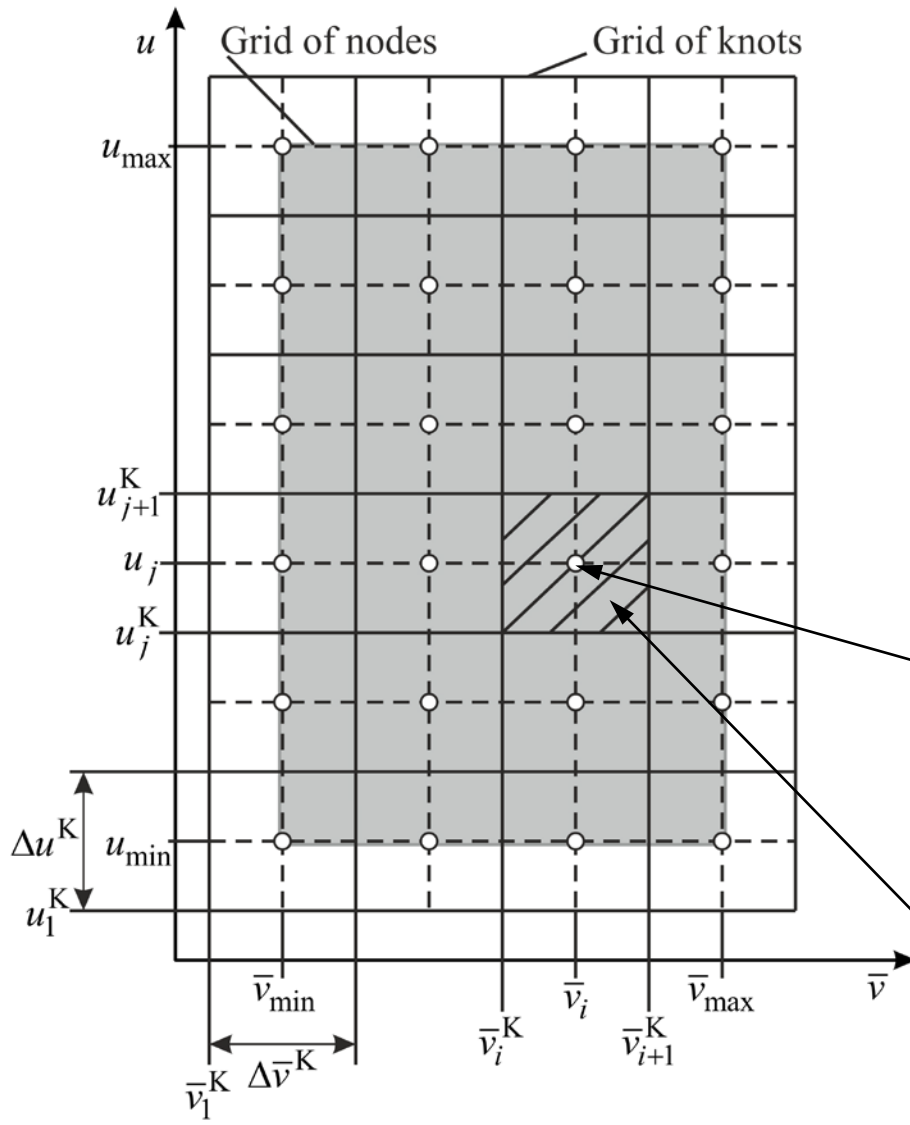
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The Spline-Based Table Look-up Method (SBTL)

Generation of a Spline-Function $p^{\text{SPL}}(\bar{v}, u)$ based on the IAPWS-95:



- **Variable transformations ($v \rightarrow \bar{v}$):**
 - Transformation of an irregularly shaped domain into a rectangle
 - Linearization
→ Accuracy enhancement
- **Generation of a rectangular grid of nodes:**
 - I nodes along \bar{v} and J nodes along u (preferably equidistant for simplified cell search)
 - Calculation of node values
- **Cell-definition in the grid of knots:**
 - Spline polynomial:

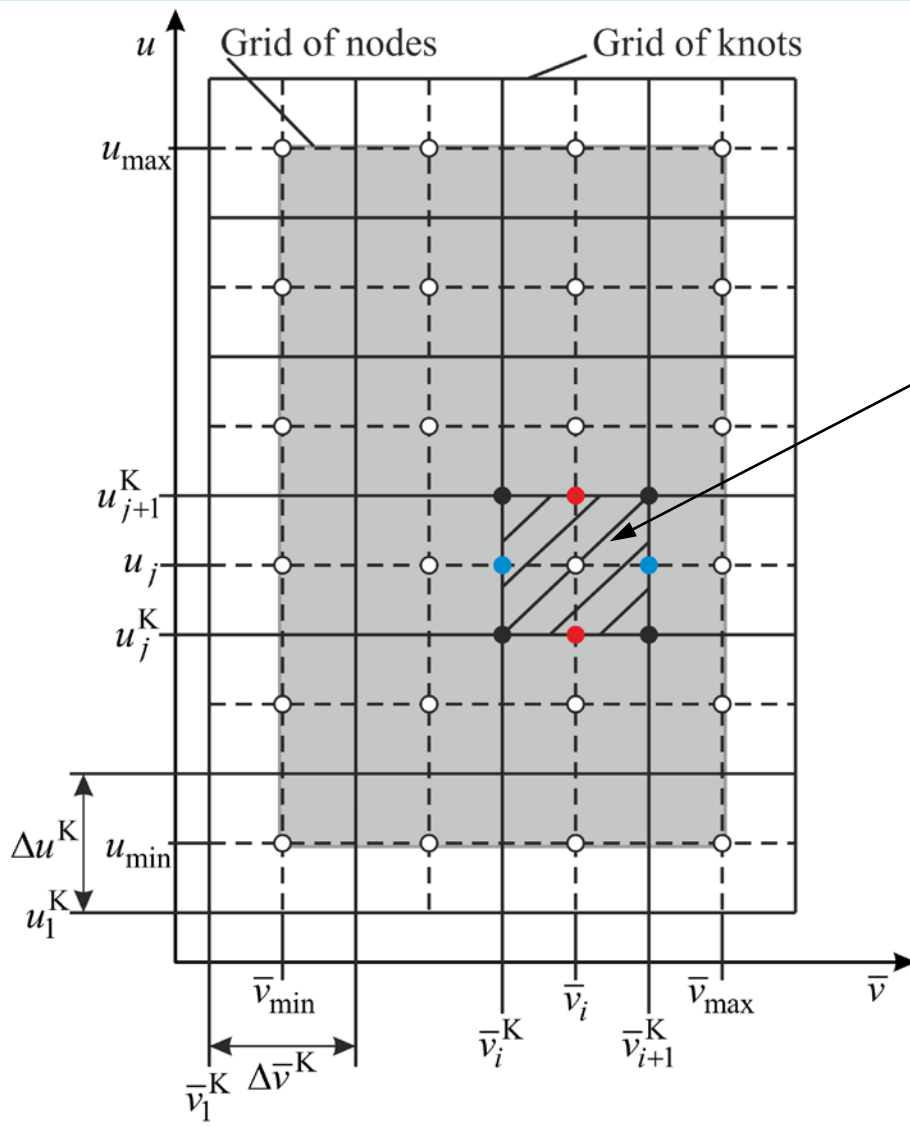
$$p_{i,j}(\bar{v}_i, u_j) = p^{\text{EOS}}(\bar{v}(v_i), u_j)$$

from the underlying equation of state $p^{\text{EOS}}(v, u)$

$$p_{\{i,j\}}^{\text{SPL}}(\bar{v}, u) = \sum_{k=1}^3 \sum_{l=1}^3 a_{ijkl} (\bar{v} - \bar{v}_i)^{k-1} (u - u_j)^{l-1}$$

The Spline-Based Table Look-up Method (SBTL)

Generation of a Spline-Function $p^{\text{SPL}}(\bar{v}, u)$ based on the IAPWS-95:



- Variable transformations ($v \rightarrow \bar{v}$)
- Generation of a rectangular grid of nodes
- Cell-definition in the grid of knots:
 - Spline polynomial:

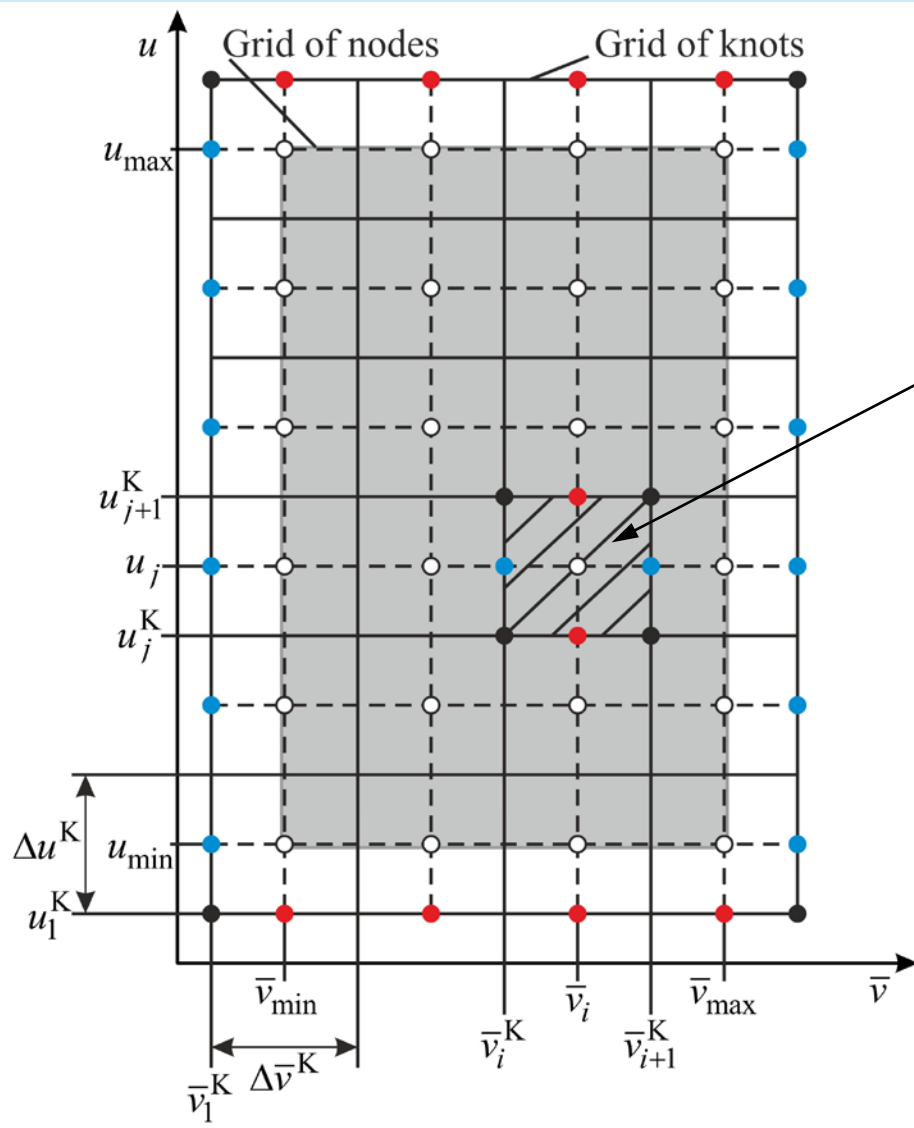
$$p_{\{i,j\}}^{\text{SPL}}(\bar{v}, u) = \sum_{k=1}^3 \sum_{l=1}^3 a_{ijkl} (\bar{v} - \bar{v}_i)^{k-1} (u - u_j)^{l-1}$$

- Calculation of all $9IJ$ spline coeff. a_{ijkl} from:
 - IJ interpolation conditions:
 - $p_{\{i,j\}}^{\text{SPL}}(\bar{v}_i, u_j) = p_{i,j}(\bar{v}_i, u_j)$
 - $3IJ+2(I+J)+1$ unknown derivatives:

$$\bullet \left(\frac{\partial p}{\partial \bar{v}} \right)_u \quad \bullet \left(\frac{\partial p}{\partial u} \right)_{\bar{v}} \quad \bullet \left(\frac{\partial^2 p}{\partial \bar{v} \partial u} \right)$$

The Spline-Based Table Look-up Method (SBTL)

Generation of a Spline-Function $p^{\text{SPL}}(\bar{v}, u)$ based on the IAPWS-95:



- Variable transformations ($v \rightarrow \bar{v}$)
- Generation of a rectangular grid of nodes
- Cell-definition in the grid of knots:
 - Spline polynomial:

$$p_{\{i,j\}}^{\text{SPL}}(\bar{v}, u) = \sum_{k=1}^3 \sum_{l=1}^3 a_{ijkl} (\bar{v} - \bar{v}_i)^{k-1} (u - u_j)^{l-1}$$

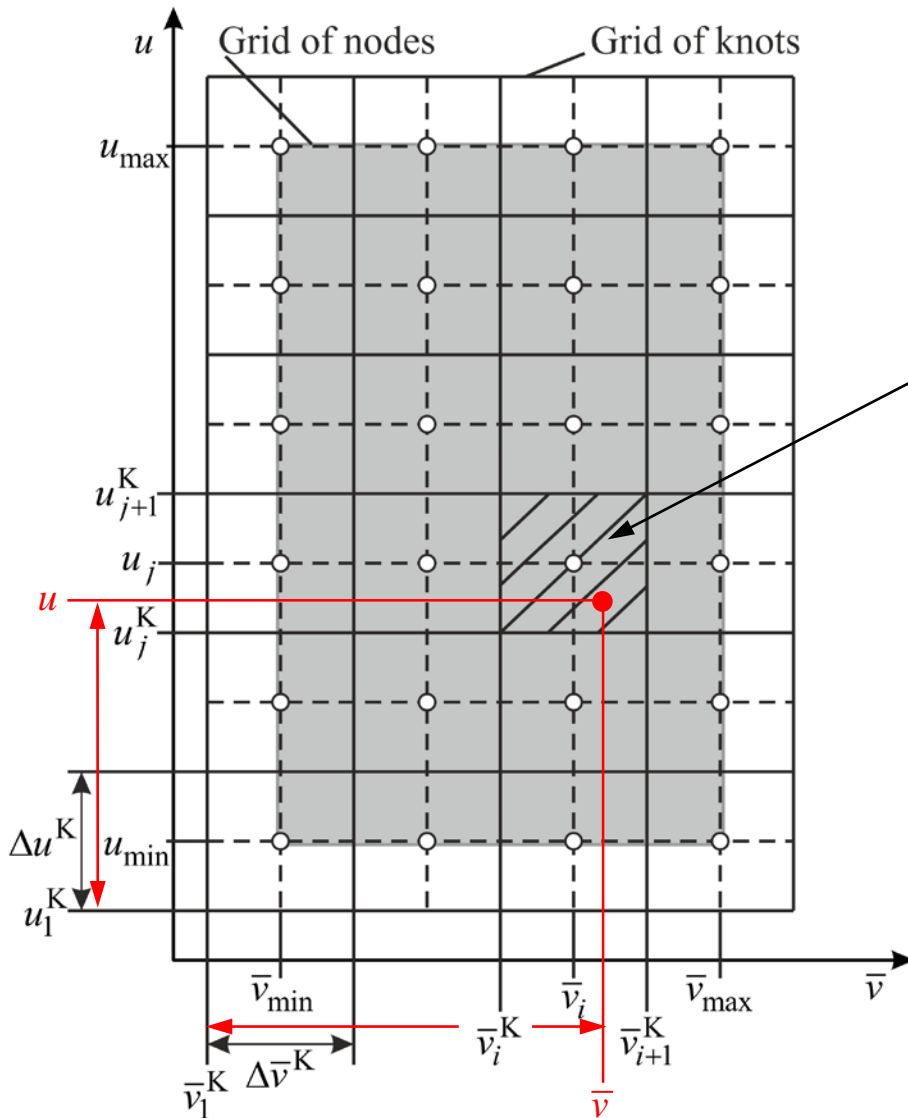
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$$\bullet \left(\frac{\partial p}{\partial \bar{v}} \right)_u \quad \bullet \left(\frac{\partial p}{\partial u} \right)_{\bar{v}} \quad \bullet \left(\frac{\partial^2 p}{\partial \bar{v} \partial u} \right)$$

When these $2(I+J)+4$ derivatives are given at the outer boundary of the grid of knots, the remaining unknowns can be obtained from $3(IJ-1)$ equations, so that the **resulting spline function is continuously differentiable once.**

The Spline-Based Table Look-up Method (SBTL)

Generation of a Spline-Function $p^{\text{SPL}}(\bar{v}, u)$ based on the IAPWS-95:



- Variable transformations ($v \rightarrow \bar{v}$)
- Generation of a rectangular grid of nodes
- Cell-definition in the grid of knots:
 - Spline polynomial:

$$p_{\{i,j\}}^{\text{SPL}}(\bar{v}, u) = \sum_{k=1}^3 \sum_{l=1}^3 a_{ijkl} (\bar{v} - \bar{v}_i)^{k-1} (u - u_j)^{l-1}$$

- Calculation of all $9IJ$ spline coeff. a_{ijkl} :
- Grid optimization:
 - for the desired accuracy
 - for minimal computing times
 - for minimal number of nodes (data)
- Grid of knots along with all spline coeff. is stored for property calculations
- Evaluation of the SBTL property function:
 - Transformation of $v \rightarrow \bar{v}$
 - Fast cell determination $\{i, j\}$:

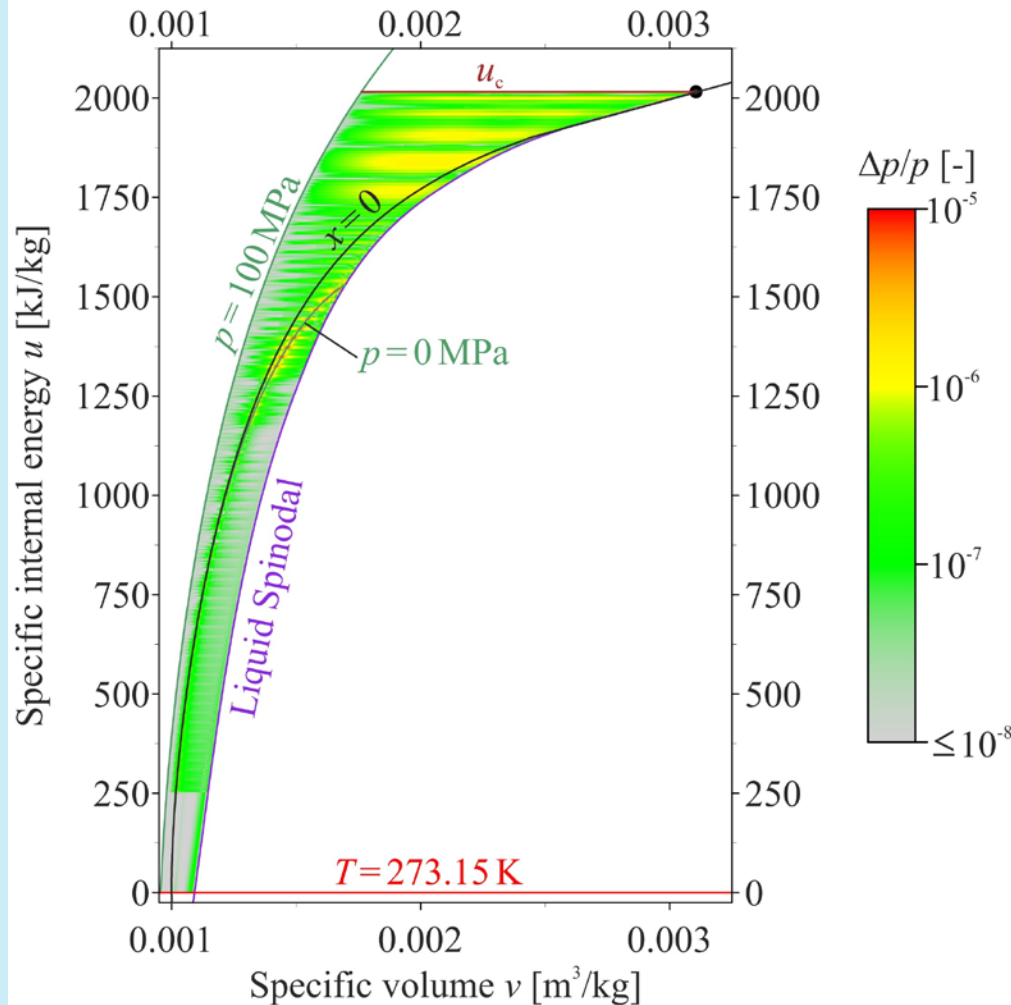
$$i = \text{floor} \left(\frac{\bar{v} - \bar{v}_1^K}{\Delta \bar{v}^K} \right)$$

$$j = \text{floor} \left(\frac{u - u_1^K}{\Delta u^K} \right)$$

- Evaluation of the polynomial $p_{\{i,j\}}^{\text{SPL}}(\bar{v}, u)$

The Spline-Based Table Look-up Method (SBTL)

SBTL function $p^L(v,u)$ – Deviations from IAPWS-95:



$$\bar{v}(v,u) = \frac{\bar{v}_{\max} - \bar{v}_{\min}}{v_{\text{liq.-spinodal}}(u) - v_{p_{\max}}(u)} (v - v_{p_{\max}}(u)) + \bar{v}_{\min}$$

Grid of nodes:

$v \text{ [m}^3\text{/kg]} \rightarrow \bar{v}(v,u)$	
domain	grid lines
$1 \leq \bar{v} \leq 20$	50
$20 \leq \bar{v} \leq 40$	150
$40 \leq \bar{v} \leq 95$	75
$95 \leq \bar{v} \leq 100$	75
$u \text{ [kJ/kg]}$	
domain	grid lines
$-8 \leq u \leq 250$	350
$250 \leq u \leq 2040$	225

Maximum deviations from IAPWS-95 in the stable single-phase region:

$$p \leq 2.5 \text{ MPa: } \left| \Delta p / p \right|_{\max} = 0.092 \%$$

$$p > 2.5 \text{ MPa: } \left| \Delta p \right|_{\max} = 2.74 \text{ kPa}$$

The Spline-Based Table Look-up Method (SBTL)

SBTL function $p^G(v,u)$ – Deviations from IAPWS-95:

Grid of nodes:

$$v \text{ [m}^3\text{/kg]} \rightarrow \bar{v}(v) = \ln(v)$$

domain

grid lines

$$\bar{v}(1.6 \times 10^{-3}) \leq \bar{v} \leq \bar{v}(8 \times 10^{-3})$$

100

$$\bar{v}(8 \times 10^{-3}) \leq \bar{v} \leq \bar{v}(1189)$$

200

$$u \text{ [kJ/kg]}$$

domain

grid lines

$$1845 \leq u \leq 2650$$

100

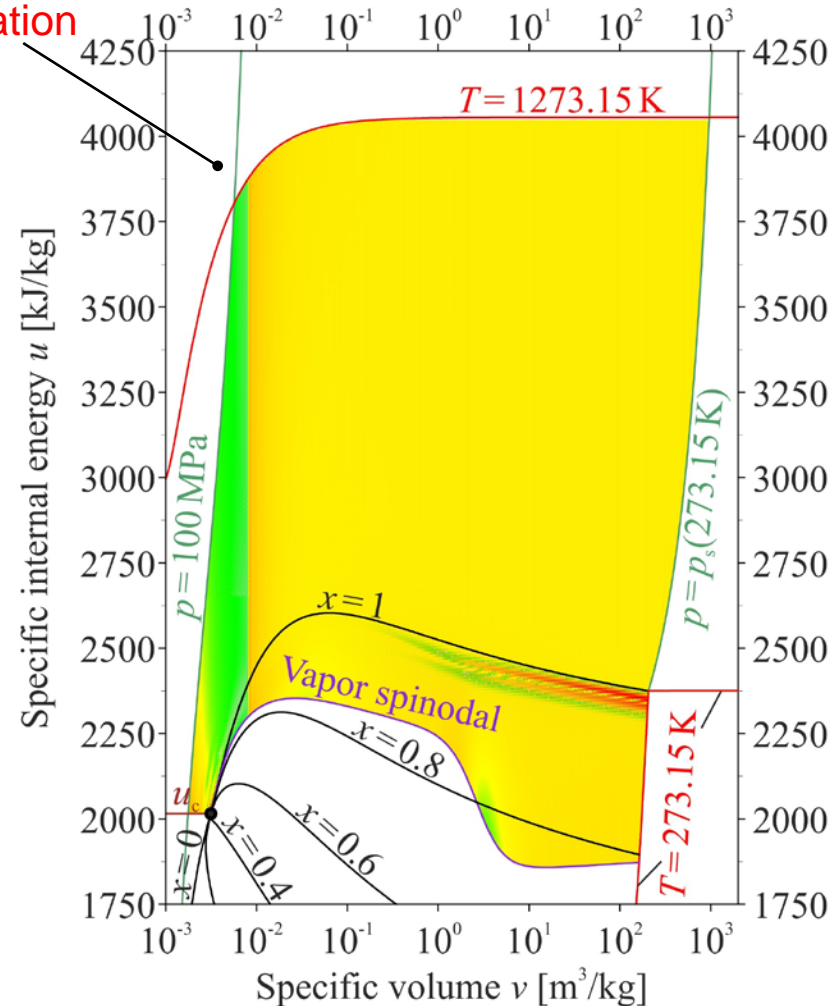
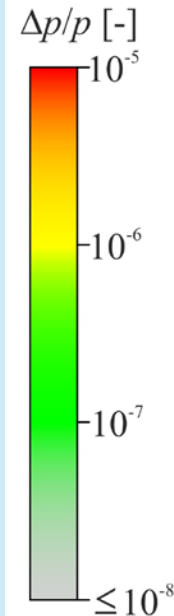
$$2650 \leq u \leq 4085$$

100

Maximum deviations from IAPWS-95 in the stable single-phase region:

$$\left| \Delta p / p \right|_{\max} = 0.001 \%$$

Extrapolation
of EOS



$$\bar{v}(v) = \ln(v)$$

The Spline-Based Table Look-up Method (SBTL)

Calculation of inverse spline functions (Example: bi-quadratic polynomial):

Spline function:

$$p_{\{i,j\}}^{\text{SPL}}(\bar{v}, u) = \sum_{k=1}^3 \sum_{l=1}^3 a_{ijkl} (\bar{v} - \bar{v}_i)^{k-1} (u - u_j)^{l-1}$$

Inverse spline function:

$$u_{\{i,j\}}^{\text{INV}}(p, \bar{v}) = \frac{(-B \pm \sqrt{B^2 - 4AC})}{2A} + u_j$$

with

$$A = a_{ij13} + \Delta\bar{v}_i (a_{ij23} + a_{ij33}\Delta\bar{v}_i)$$

$$B = a_{ij12} + \Delta\bar{v}_i (a_{ij22} + a_{ij32}\Delta\bar{v}_i)$$

$$C = a_{ij11} + \Delta\bar{v}_i (a_{ij21} + a_{ij31}\Delta\bar{v}_i) - p$$

and

$$\Delta\bar{v}_i = (\bar{v} - \bar{v}_i)$$

$$(\pm) = \text{sign}(B)$$

Auxiliary spline function for fast determination of cell $\{i,j\}$: $u_{\{i,j\}}^{\text{AUX}}(p, \bar{v})$

⇒ The inverse spline function $u(p, v)$ is numerically consistent with $p(v, u)$.

⇒ The inverse spline function does not require extensive iterative algorithms.

The Spline-Based Table Look-up Method (SBTL)

Application of inverse spline functions:

Spline functions of (v,u) :

(p,v) :

(u,s) :

Pressure

$$p^{\text{SPL}}(v,u) \xrightarrow{\text{red arrow}} u^{\text{INV}}(p,v)$$

Temperature

$$T^{\text{SPL}}(v,u) \quad T^{\text{SPL}}, s^{\text{SPL}}, w^{\text{SPL}}, \eta^{\text{SPL}}, \lambda^{\text{SPL}}(v, u^{\text{INV}})$$

Spec. Entropy

$$s^{\text{SPL}}(v,u) \xrightarrow{\text{red arrow}} v^{\text{INV}}(u,s)$$

Speed of sound

$$w^{\text{SPL}}(v,u) \quad p^{\text{SPL}}, T^{\text{SPL}}, w^{\text{SPL}}, \eta^{\text{SPL}}, \lambda^{\text{SPL}}(v^{\text{INV}}, u)$$

Dynamic Viscosity

$$\eta^{\text{SPL}}(v,u)$$

Thermal conductivity

$$\lambda^{\text{SPL}}(v,u)$$

⇒ Property functions are numerically consistent with each other.

⇒ All thermodynamic and transport properties including derivatives and inverse functions are calculated without iterations.

IAPWS Guideline on the Fast Calculation of Steam and Water Properties With the Spline-Based Table Look-Up Method (SBTL)

Contents:

- **Fundamentals of the SBTL method**
- **SBTL property functions based on IAPWS-IF97**
 - SBTL functions of (v,u) with inverse Functions of (p,v) and (u,s)
 - SBTL functions of (p,h) with inverse Functions of (p,T) , (p,s) and (h,s)
 - SBTL functions for the gas phase including the metastable region
- **SBTL property functions based on IAPWS-95**
 - SBTL functions of (v,u)
 - SBTL functions of (p,h)
- **Results of the evaluation:**
 - Deviations from the underlying IAPWS formulations
 - Computing-Time Ratios with regard to the underlying IAPWS formulations
 - Application of the SBTL method in extensive process simulations:
 - Computational Fluid Dynamics (CFD)
 - Heat Cycle Calculations

Deviations SBTL Property Functions from underlying IAPWS Formulations

Permissible deviations:

SBTL functions	Region		
	liquid	vapor	two-phase
$p(v,u)$	$p \leq 2.5$ MPa: 0.6 %	0.001 %	0.0035 %
	$p > 2.5$ MPa: 15 kPa		
$T(v,u), T(p,h)$	1 mK	1 mK	1 mK
$v(p,h)$	0.001 %	0.001 %	0.001 %
$s(v,u), s(p,h)$	10^{-6} kJ/(kg K)	10^{-6} kJ/(kg K)	10^{-6} kJ/(kg K)
$w(v,u), w(p,h)$	0.001 %	0.001 %	-
$\eta(v,u), \eta(p,h)$	0.001 %	0.001 %	-

Deviations with regard to IAPWS-95 near the critical point:

$$|p_{\max}| < 0.01 \%, \quad |T_{\max}| < 10 \text{ mK}, \quad |v_{\max}| < 0.03 \%, \quad |w_{\max}| < 5 \%$$

➤ The deviations of the SBTL functions with regard to the current IAPWS formulations are within the permissible values!

SBTL functions of (v,u) and inverse functions of (p,v) and (u,s) – Computing times in comparison with IAPWS-IF97 and IAPWS-95

Computing-Time Ratio $CTR = \frac{\text{Computing time of the IAPWS - IF97 (BWE) function}}{\text{Computing time of the SBTL function}}$

SBTL function	IAPWS-IF97 region				
	1 (liquid)	2 (vapor)	3 (critical)	4 (two-phase)	5 (high-temp.)
$p(v,u)$	130	271	161	19.6	470
$T(v,u)$	161	250	158	20.6	442
$u(p,v)$	2.0	6.4	2.8	5.6	3.2
$v(u,s)$	43.5	66.4	78.8	16.2	134
$T(p,h)$	17.2 (2.9 ^a)	23.9 (4.7 ^a)	585 (3.0 ^a)	18.1	53.4
$v(p,h)$	18.7 (3.8 ^a)	23.5 (6.1 ^a)	671 (5.1 ^a)	5.5	46.1

a) Backward equation with limited numerical consistency, e.g. $T(p,h)$ and $h(p,T)$

IAPWS-IF97: Extended Steam Tables Software (includes determination of IAPWS-IF97 regions)

IAPWS-95: REFPROP (internal, highly optimized functions (phase is known))

Processor: Intel Xeon – 3,2GHz

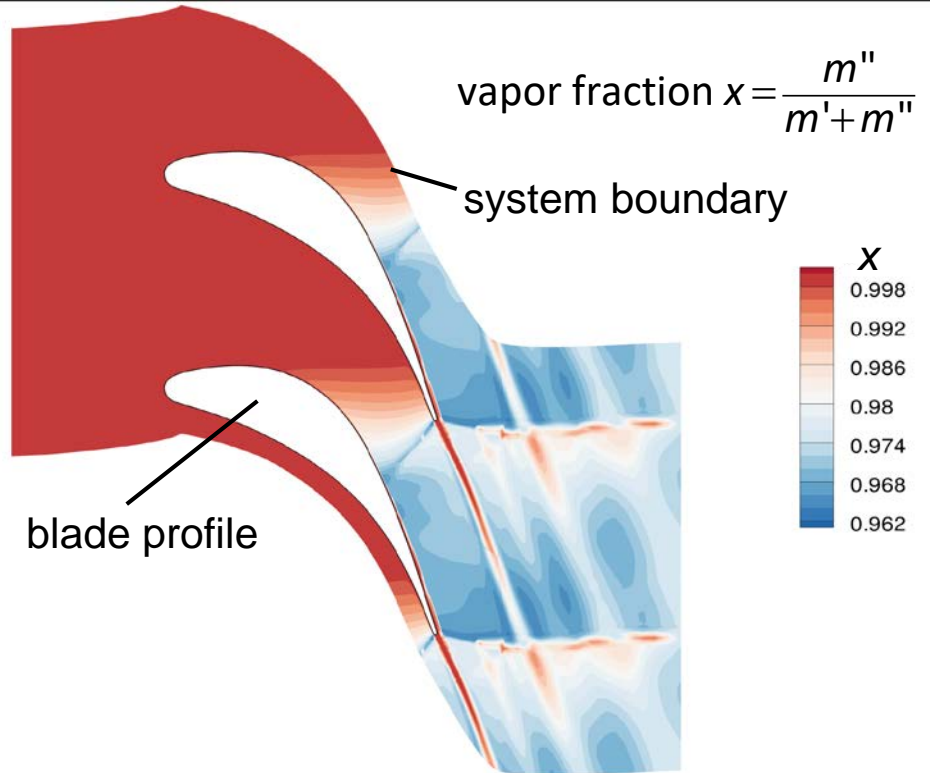
OS: Windows7 (32 Bit)

Compiler: Intel Composer XE 2011

➤ **SBTL functions are up to 270 (15000) times faster!**

Application of the SBTL Method in Computational Fluid Dynamics (CFD)

Simulation of condensing steam flow around a fixed blade (White et al.)



German Aerospace Center (DLR)
Institute of Propulsion Technology
Numerical Methods,
Cologne, Germany

CFD-Software TRACE (DLR)

- **Implementation of SBTL property functions based on IAPWS-IF97**
- **Extensive assessment of simulation results, computing times, and convergence behavior**

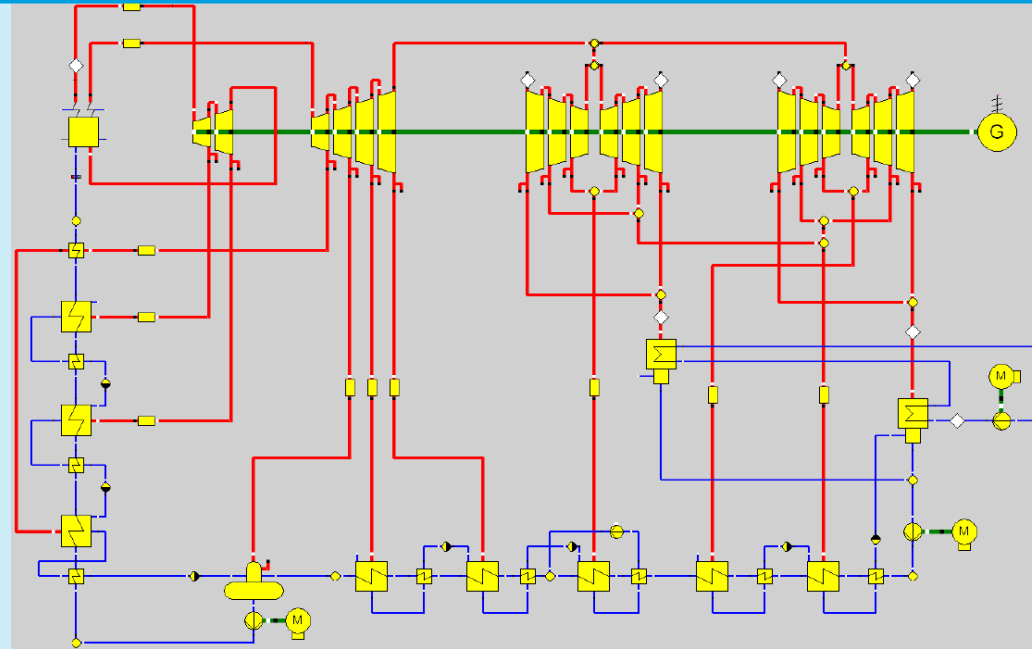
Application of the SBTL method:

- **in comparison with the direct application of IAPWS-IF97:**
 - the computing times are reduced by factors of 6 - 10!
 - the differences in the results of the process simulations are negligible!
- **in comparison with the application of the ideal gas-model:**
 - the computing times are increased by a factor of 1.4 only!

Application of the SBTL Method in Heat Cycle Calculations

➤ **EBSILON Professional**
(STEAG Energy Services)

➤ **KRAWAL-modular**
(SIEMENS PG)



Implementation of SBTL property functions of (p,h) and the corresponding inverse functions of (p,T) , (p,s) , and (h,s) based on IAPWS-IF97:

- ➡ For a typical simulation of a conventional power plant in EBSILON Professional, the share of property calculations in the overall computing time is reduced from 25 % to 8 % and the overall computing time is reduced by 17 %.
- ➡ In KRAWAL-modular, the overall computing times are reduced on average by 50 %.
- ➡ The differences in the results of the process simulations are negligible ($< 0.02 \%$).

Further Applications of the SBTL Method

➤ Fluids:

- Helium
- Nitrogen
- Carbon dioxide
- Humid air

➤ Software:

- **RELAP-7** (Reactor Excursion and Leak Analysis Program), developed at the Idaho National Laboratory (INL), USA
- **ATHLET** (Analysis of Thermal-hydraulics of Leaks and Transients), developed at the GRS, Germany
- **SubChanFlow** and **TwoPorFlow** (thermal-hydraulic codes), developed at KIT, Germany
- **DYNAPLANT** (simulation of non-stationary processes in power plants), developed by SIEMENS, PG, Germany

Summary

- **Spline-Based Table Look-Up Method (SBTL) – a supplement to existing standards:**
 - Reproduces existing standards with high accuracy at high computing speed
 - Inverse spline functions are numerically consistent with their forward functions
 - Property functions and their first derivatives are continuous
- **SBTL functions based on IAPWS-IF97 and IAPWS-95:**
 - Property functions of IAPWS Standards are reproduced with an accuracy of 10 – 100 ppm
 - Computing speeds are considerably increased
(SBTL functions of (v,u) up to 270 times faster than IAPWS-IF97)
- **Method has been tested in extensive process simulations:**
 - in Computational Fluid Dynamics (CFD):
 - enables consideration of the real fluid behavior with high accuracy
 - 6-10 times faster than simulations with IAPWS-IF97
 - with regard to the ideal-gas model, the computing times increase by 40 % only
 - in Heat Cycle Calculations:
 - with regard to the direct application of IAPWS-IF97, the overall computing times are considerably reduced (17 % - 50 %)
 - the differences in the results of the process simulations are negligible

The International Association for the Properties of Water and Steam

Stockholm, Sweden

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President:

Dr. David Guzonas

Canadian Nuclear Laboratories
Chalk River, Ontario, Canada

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Thank you for your attention!