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Supplementary Backward Equations for the Industrial Formulation IAPWS-IF97 of Water and Steam for Fast Calculations of Heat Cycles, Boilers, and Steam Turbines

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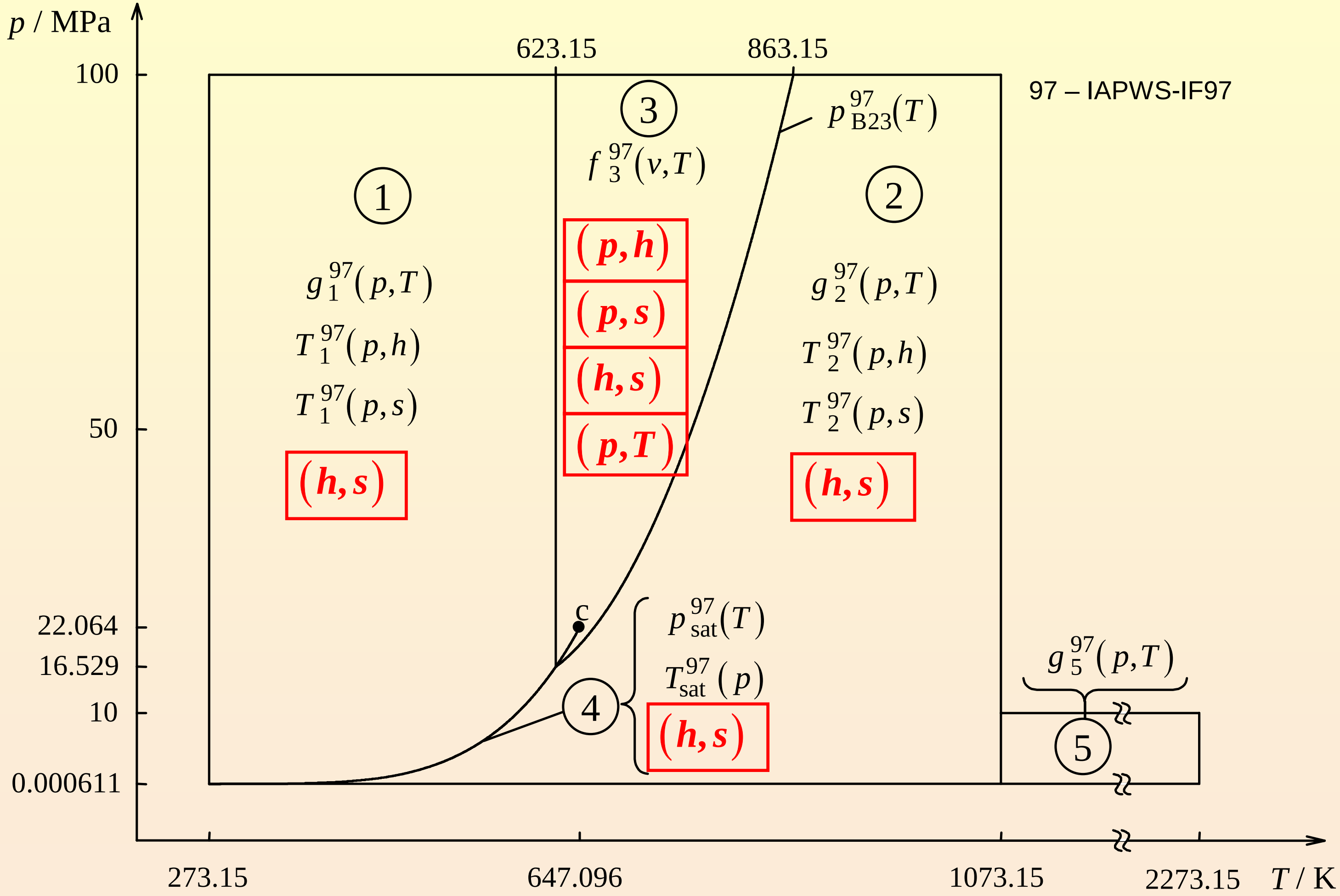
IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam IAPWS-IF97

Result of international surveys in industry, organized by IAPWS



Requirements on Backward Equations

1. Extremely high numerical consistency

→ Deviation between the backward equation and the relating fundamental equation

Example: Backward equations $T(p,h)$

$$|\Delta T| = |T - T(p,h)|$$

where $h(p,T)$ – derived from the fundamental equation $g(p,T)$

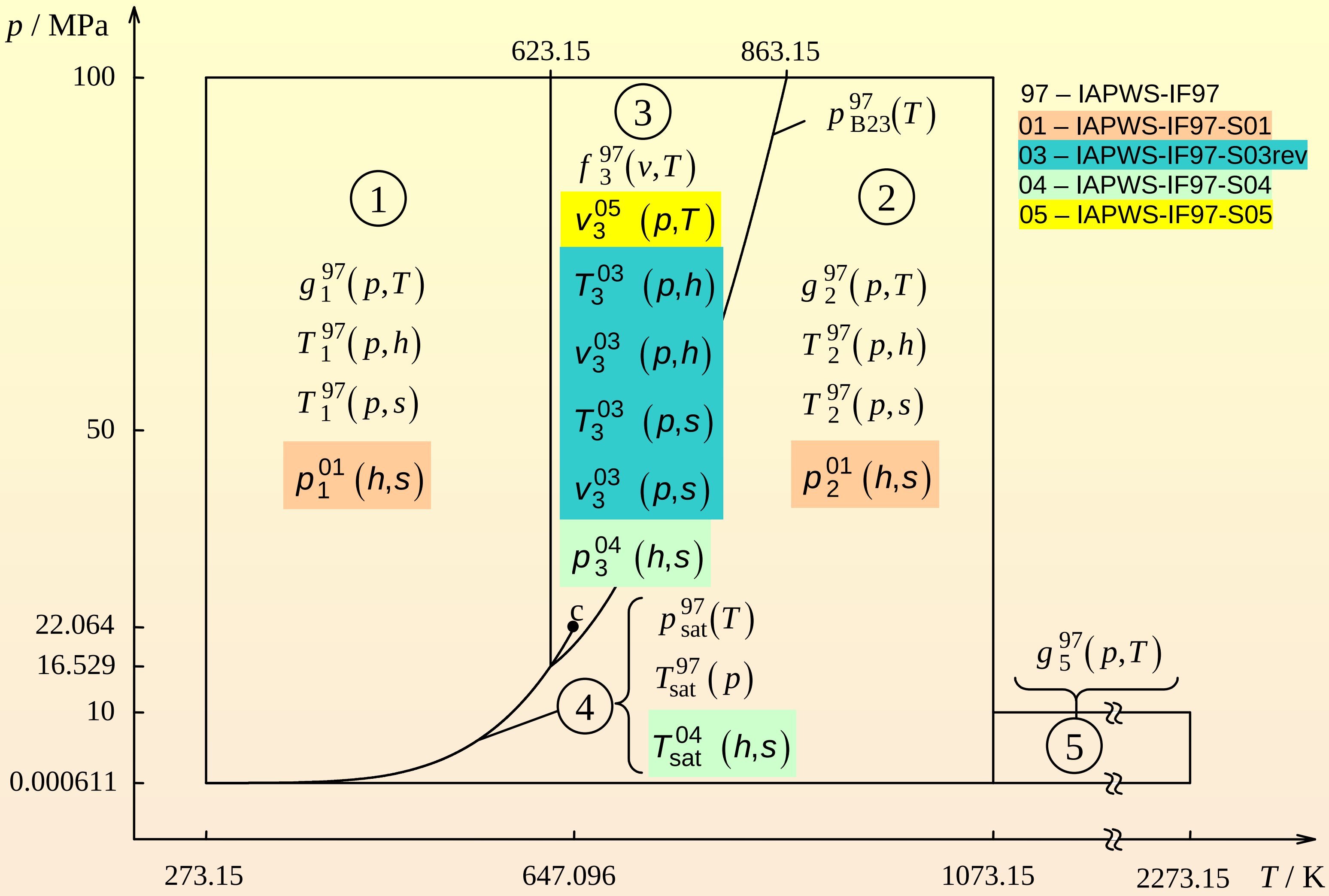
→ Corresponds to iteration accuracy otherwise used in numerical calculations of process modeling

→ Determined by IAPWS based on an international survey in industry

Problem: The numerical consistency is more than one magnitude higher then accuracy of the properties themselfs

2. Calculation of the backward equations should be much faster than the corresponding iterations of the fundamental equations

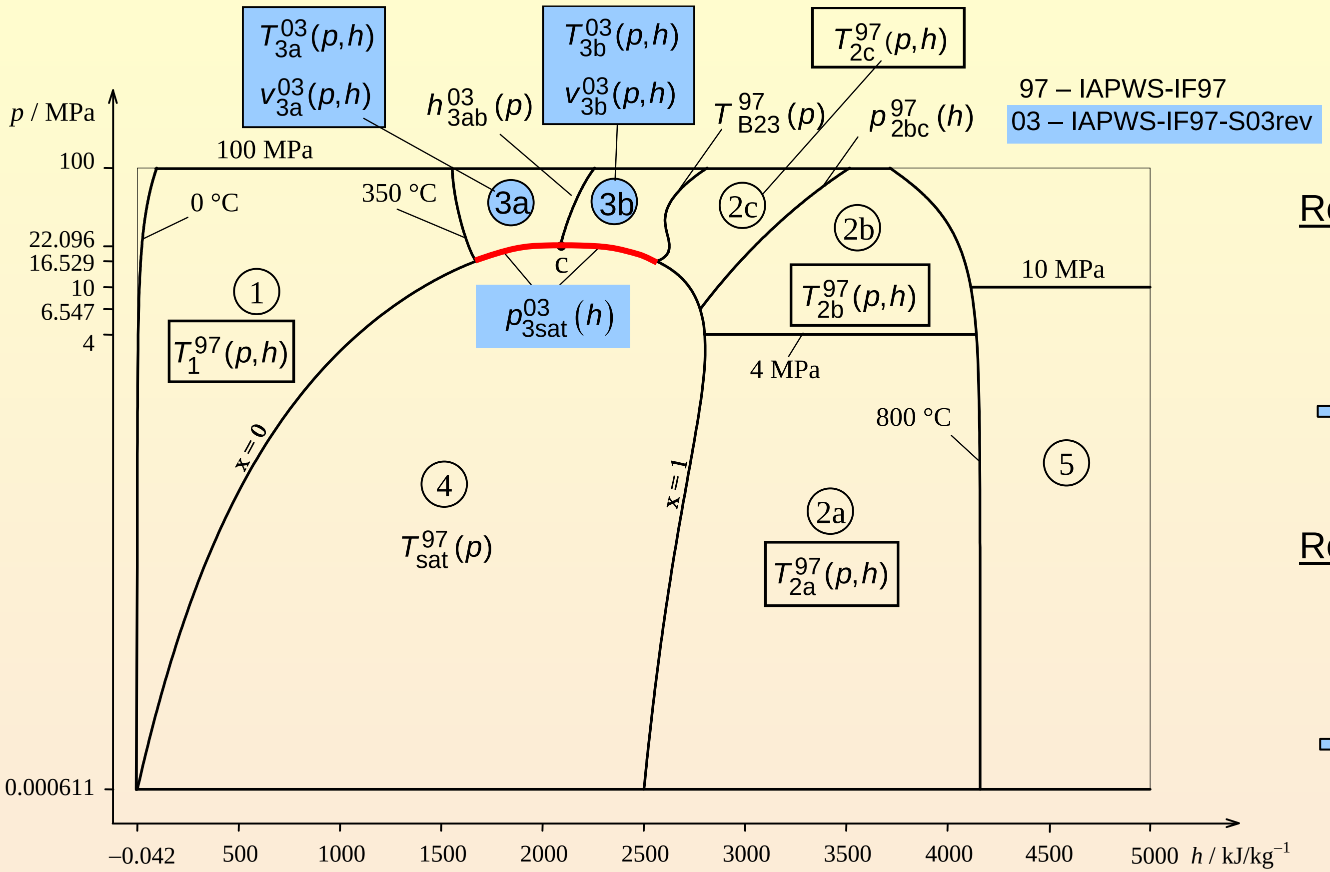
Supplementary Backward Equations for IAPWS-IF97

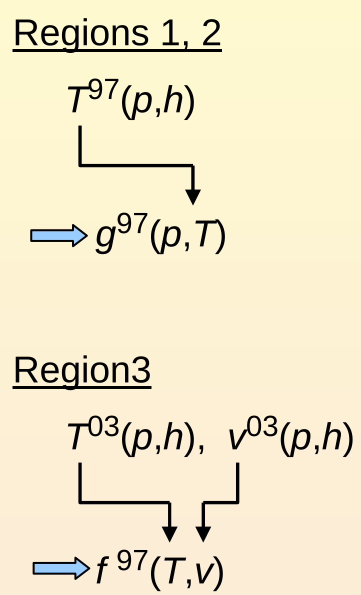


Supplementary Release	Equations	Status
IAPWS-IF97-S01: Supplementary Release on Backward Equations for Pressure as a Function of Enthalpy and Entropy $p(h,s)$ to the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam.	$p_1(h,s)$ $p_2(h,s)$	adopted in 2001
IAPWS-IF97-S03rev: Revised Supplementary Release on Backward Equations for the Functions $T(p,h)$, $v(p,h)$ and $T(p,s)$, $v(p,s)$ for Region 3 of the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam.	$T_3,v_3(p,h)$ $T_3,v_3(p,s)$ $p_{3sat}(h)$ $p_{3sat}(s)$	adopted in 2003 revised in 2004
IAPWS-IF97-S04: Supplementary Release on Backward Equations $p(h,s)$ for Region 3, Equations as a Function of h and s for the Region Boundaries, and an Equation $T_{sat}(h,s)$ for Region 4 of the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam.	$p_3(h,s)$ $T_{sat}(h,s)$ $h'(s)$, $h''(s)$ $h_{B13}(s)$ $T_{B23}(h,s)$	adopted in 2004
IAPWS-IF97-S05: Supplementary Release on Backward Equations for Specific Volume as a Function of Pressure and Temperature $v(p,T)$ for Region 3 of the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam.	$v_3(p,T)$	adopted in 2005

Further information concerning supplementary releases or other releases issued by IAPWS can be obtained from http://www.iapws.org.

Backward and Boundary Equations for Functions of Pressure and Enthalpy (*p*,*h*)





Backward Equations $T(p,h)$ and $v(p,h)$

Structure

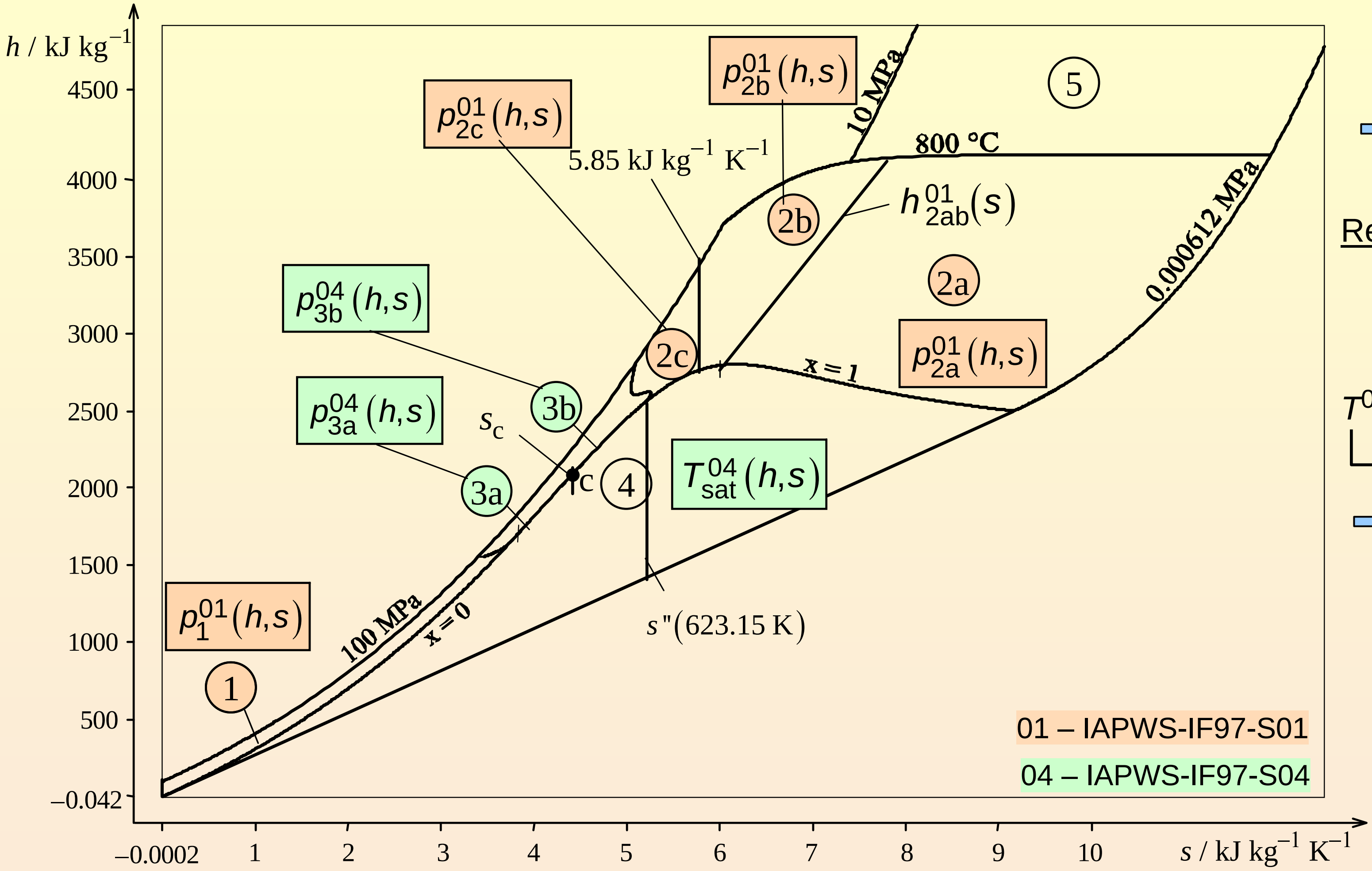
$$\frac{T(p,h)}{T^*} = \sum_{i=1}^{N} n_i \left(\frac{p}{p^*}+a\right)^{I_i} \left(\frac{h}{h^*}+b\right)^{J_i} \qquad \frac{v(p,h)}{v^*} = \sum_{i=1}^{N} n_i \left(\frac{p}{p^*}+a\right)^{I_i} \left(\frac{h}{h^*}+b\right)^{J_i}$$

Numerical consistency

Equation	N	a	b	$\lvert\Delta T\rvert_{\mathrm{tol}}$ mK	$\lvert\Delta T\rvert_{\mathrm{max}}$ mK
$T_1^{97}(p,h)$	20	0	1	25	23.6
$T_{2a}^{97}(p,h)$	34	0	–2.1	10	9.3
$T_{2b}^{97}(p,h)$	38	–2	–2.6	10	9.6
$T_{2c}^{97}(p,h)$	23	25	–1.8	25	23.7
$T_{3a}^{03}(p,h)$	31	0.24	–0.615	25	23.6
$T_{3b}^{03}(p,h)$	33	0.298	–0.720	25	19.6

Equation	N	a	b	$\lvert\Delta v/v\rvert_{\mathrm{tol}}$ %	$\lvert\Delta v/v\rvert_{\mathrm{max}}$ %
$v_{3a}^{03}(p,h)$	32	0.128	–0.727	0.01	0.0080
$v_{3b}^{03}(p,h)$	30	0.0661	–0.72	0.01	0.0095

Backward Equations for Functions of Enthalpy and Entropy (*h*,*s*)



Regions 1, 2

$p^{01}(h,s)$

$T^{97}(p,h)$

$\Rightarrow g^{97}(T,p)$

Region 3

$p^{04}(h,s)$,

$T^{03}(p,h)$, $v^{03}(p,s)$

$\Rightarrow f^{97}(T,v)$

Region 4

$T^{04}_{sat}(h,s)$

$p^{97}_{sat}(T_{sat})$

$$x = \frac{h - h'}{h'' - h'}$$

Backward Equations $p(h,s)$ and $T_{sat}(h,s)$

Structure

$$\frac{p(h,s)}{p^*} = \left[\sum_{i=1}^{N} n_i \left(\frac{h}{h^*} + a \right)^{I_i} \left(\frac{s}{s^*} + b \right)^{J_i} \right]^c$$

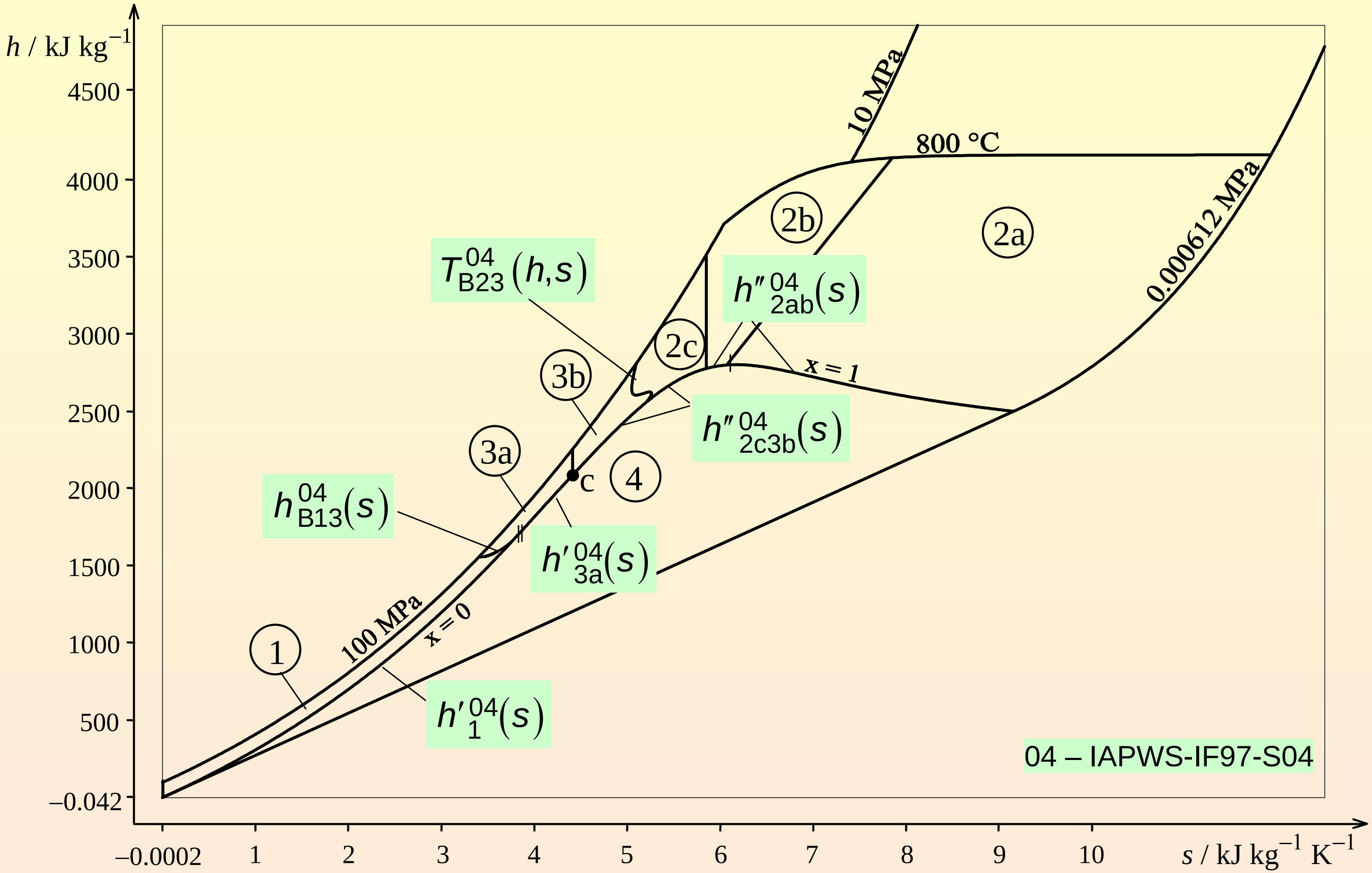
$$\frac{T_{sat}(h,s)}{T^*} = \sum_{i=1}^{36} n_i \left(\frac{h}{h^*} - 0.119 \right)^{I_i} \left(\frac{s}{s^*} - 1.07 \right)^{J_i}$$

Numerical consistency

Equation	$\lvert\Delta p/p\rvert_{tol}$ %	$\lvert\Delta p/p\rvert_{max}$ %	$\lvert\Delta T\rvert_{tol}$ mK	$\lvert\Delta T\rvert_{max}$ mK	$\lvert\Delta v/v\rvert_{tol}$ %	$\lvert\Delta v/v\rvert_{max}$ %
$p_1^{01}(h,s)$ $p \le 2.5$MPa	0.60	0.55	25	24.0		
$p > 2.5$MPa	15 kPa	14 kPa				
$p_{2a}^{01}(h,s)$	0.0035	0.0029	10	9.7		
$p_{2b}^{01}(h,s)$	0.0035	0.0034	10	9.8		
$p_{2c}^{01}(h,s)$	0.0088	0.0063	25	24.9		
$p_{3a}^{04}(h,s)$	0.01	0.0070	25	23.7	0.01	0.0097
$p_{3b}^{04}(h,s)$	0.01	0.0084	25	22.4	0.01	0.0095

Equation	$\lvert\Delta T\rvert_{tol}$ mK	$\lvert\Delta T\rvert_{max}$ mK	$\lvert\Delta p/p\rvert_{tol}$ %	$\lvert\Delta p/p\rvert_{max}$ %	$\lvert\Delta x\rvert_{tol}$ –	$\lvert\Delta x\rvert_{max}$ –
$T_{sat}^{04}(h,s)$ $s \le 5.85$ kJ kg^{-1}K^{-1}	25	0.86	0.0088	0.0034	4.4×10^{-6}	0.57×10^{-6}
$s > 5.85$ kJ kg^{-1}K^{-1}	10	0.67	0.0035	0.0029	0.64×10^{-6}	0.25×10^{-6}

Boundary Equations for Functions of Enthalpy and Entropy (h,s)



Computing Time in Comparison with IAPWS-IF97 Fundamental Equations

Computing Time Ratio (*CTR*)

$$CTR = \frac{Compting\ time\ of\ fundamental\ eq.}{Computing\ time\ of\ backward\ eq.}$$

	CTR Backward Function			
Region	(p,h)	(p,s)	(h,s)	(p,T)
① Liquid	25	38	35	-
② Vapor	11	14	46	-
③ Critical and Supercritical	14	14	10	17
④ Two-Phase	-	-	14	-

Calculations of heat cycles, boilers and steam turbines may be 2... 3 times faster when using the backward and boundary equations

Conclusions

- Backward and boundary equations for the functions of (p,h), (p,s), (h,s) and (p,T) have been developed.
- The equations were adopted as supplements to the Industrial Formulation IAPWS-IF97.
- Their numerical consistencies are sufficient for most applications in heat-cycle, boiler, and steam-turbine calculations.
- Using the equations, the properties as functions of (p,T), (p,h), (p,s), and (h,s) including determination of the region can be calculated without iterations.
- Resulting, process calculations will be between 2 and 3 times faster when using the supplementary backward and boundary equations.
- For applications where the demands on numerical consistency are extremely high, the equations can be used for calculating very accurate starting values in iterations.

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Industrial Requirements
and Solutions
(IRS)

Working Group
Thermophysical Properties
of Water and Steam
(TPWS)

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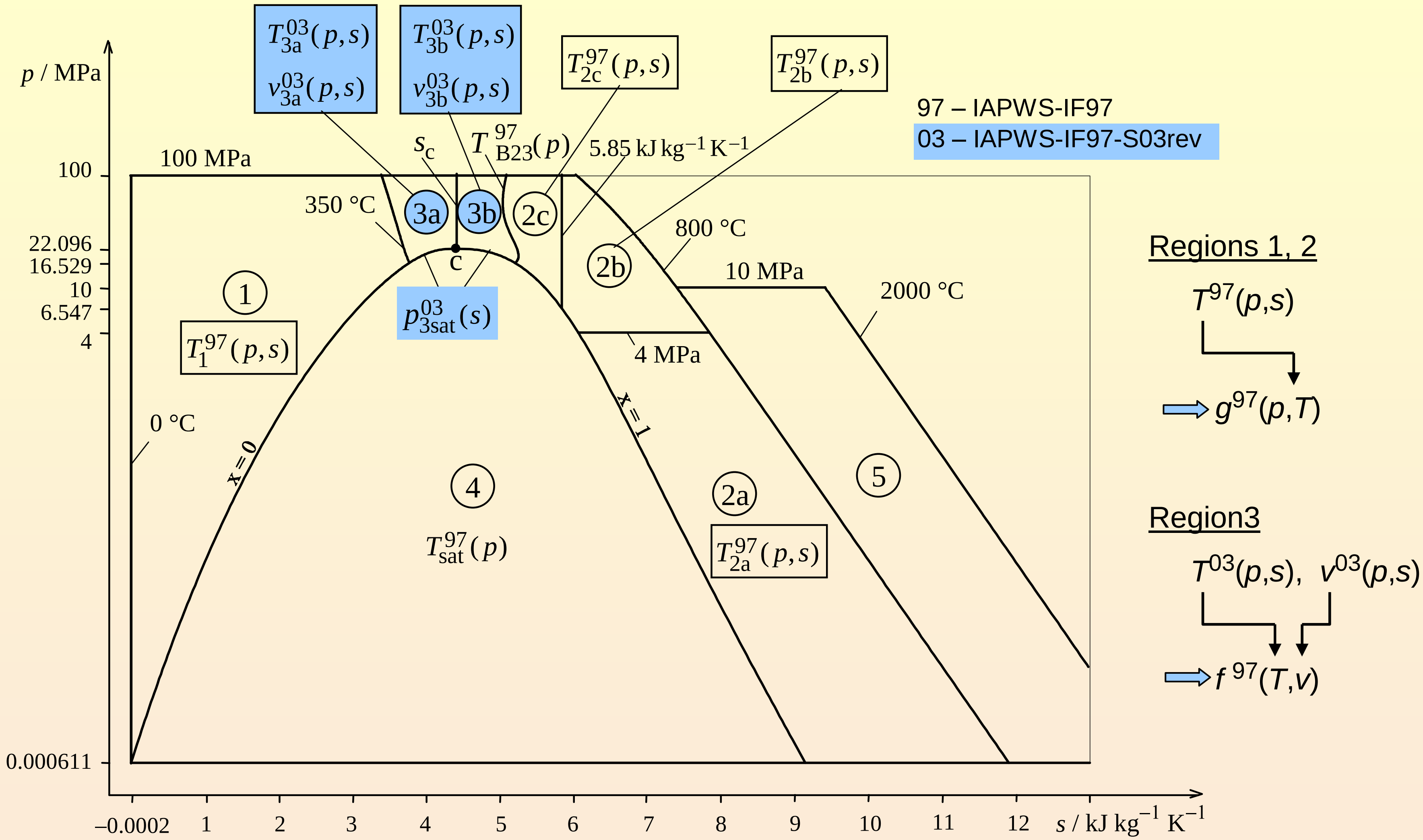
Backward and Boundary Equations for Functions of Pressure and Entropy (*p*,*s*)



Boundary Equation $h''_{2ab}(s)$

$$\frac{h''^{04}_{2ab}(s)}{2800\,\mathrm{kJ\,kg^{-1}}} = \exp\left[\sum_{i=1}^{30} n_i \left(\frac{5.21\,\mathrm{kJ\,kg^{-1}\,K^{-1}}}{s} - 0.513\right)^{I_i} \left(\frac{s}{9.2\,\mathrm{kJ\,kg^{-1}\,K^{-1}}} - 0.524\right)^{J_i}\right]$$

$$I_i = 1...36 \qquad J_i = 1...32$$

$$\frac{h''_{\text{2ab, Lemmon}}(s)}{1\,\mathrm{kJ\,kg^{-1}}} = \sum_{i=1}^{5}\left[\frac{n_i}{\left(\frac{s}{5\,\mathrm{kJ\,kg^{-1}\,K^{-1}}}\right)^{m_i}}\right]$$

$m_1 = 1.42265361816,\ m_2 = 1.56447054786, m_3 = 2.17840735677,$

$m_4 = 15.69137200495, m_5 = 15.74810408217$

$$CTR = \frac{\textit{Compting time of fundamental eq.}}{\textit{Computing time of backward eq.}}$$

Computing Time in Comparison with IAPWS-IF97 Fundamental Equations

Computing Time Ratio (*CTR*)

$$CTR = \frac{\text{Compting time of fundamental eq.}}{\text{Computing time of backward eq.}}$$

Backward Equations

Function	Reg.	Backward Equation(s)	*CTR*
(p,h)	1	$T_1^{97}(p,h)$	5
	2	$T_2^{97}(p,h)$	6
	3	$T_3^{03}(p,h)$ & $v_3^{03}(p,h)$	16
(p,s)	1	$T_1^{97}(p,s)$	6
	2	$T_2^{97}(p,s)$	7
	3	$T_3^{03}(p,s)$ & $v_3^{03}(p,s)$	18
(h,s)	1	$p_1^{01}(h,s)$ & $T_1^{97}(p,h)$	23
	2	$p_2^{01}(h,s)$ & $T_2^{97}(p,h)$	38
	3	$p_3^{04}(h,s)$ & $T_3^{03}(p,h)$ & $v_3^{03}(p,s)$	10
	4	$T_{\text{sat}}^{04}(h,s)$ & $p_{\text{sat}}^{97}(T)$ & $x = \frac{h-h'}{h''-h'}$	11
(p,T)	3	$v_3(p,T)$	5

Computing Time in Comparison with IAPWS-IF97 Fundamental Equations

Boundary Equations

Funct.	Bound.	Reg.-Reg.	Bound. Eq.	*CTR*
(p,h)	$x = 0$ $x = 1$	3 - 4	$p_{3sat}^{03}(h)$	12
(p,s)	$x = 0$ $x = 1$	3 - 4	$p_{3sat}^{03}(s)$	9
(h,s)	$x = 0$	1 - 4	$h'^{04}_{1}(s)$	24
		3 - 4	$h'^{04}_{3a}(s)$	90
	$x = 1$	2 - 4	$h''^{04}_{2ab}(s)$, $h''^{04}_{2c3b}(s)$	20
		3 - 4	$h''^{04}_{2c3b}(s)$	60
	623.15 K	1 - 3	$h_{B13}^{04}(s)$	37
	$p_{B23}^{97}(T)$	2 - 3	$T_{B23}^{04}(h,s)$, $p_{2c}^{01}(h,s)$	20

Calculation of backward functions including determination of region boundaries

➡ 5 ... 20 times faster than iteration of fundamental equations

Calculations of heat cycles, boilers and steam turbines may be 2... 3 times faster when using the backward and boudary equations