2 Backward Functions $p(h,s)$, $T(h,s)$, and $v(h,s)$ for Region 3

2.1 Consistency Requirements. The maximum permissible temperature difference between the backward function $T_3(h,s)$ and the basic equation $f_3^{97}(v,T)$ was set by IAPWS to $|\Delta T|_{tol}$ =25 mK and corresponds to that of the backward functions $T_3^{03}(p,h)$ and $T_3^{03}(p,s)$ [11]. The tolerance $|\Delta v/v|_{tol}$=0.01% for the backward function $v_3(h,s)$ corresponds to the IAPWS requirement for the backward functions $v_3^{03}(p,h)$ and $v_3^{03}(p,s)$ [5,6].

In order to fulfill the required numerical consistency of the functions $T_3(h,s)$ and $v_3(h,s)$, the maximum relative deviation between the pressure calculated from the backward equation $p_3(h,s)$ and the IAPWS-IF97 basic equation $f_3^{97}(v,T)$ must be smaller than 0.01%.

2.2 Development of the Equations. A major motivation for the development of IAPWS-IF97 and its supplementary backward equations was to reduce the time for computing thermodynamic properties. As shown previously [12], the following functional form is effective for this purpose

$$\frac{Z(X,Y)}{Z^*}=\sum_i n_i\left(\frac{X}{X^*}+a\right)^{I_i}\left(\frac{Y}{Y^*}+b\right)^{J_i} \tag{1}$$

where the reducing parameters Z^*, X^*, and Y^* are maximum values of the corresponding property within the range of validity of the equation. The shifting parameters a and b were determined by nonlinear optimization. The exponents I_i, J_i, and coefficients n_i were determined from the structure optimization method of Wagner [13] and Setzmann and Wagner [14], which chooses the optimal terms from a bank of terms with various values of I_i and J_i. The final equations were developed using the approximation algorithm developed in previous work [15–18].

In the approximation process, the backward equations were fitted to T-v-p-h or T-v-p-s values, with p, h, and s calculated from the IAPWS-IF97 basic equation $f_3^{97}(v,T)$ for values of v and T distributed over the range of validity. The critical point was set as a constraint. The computing time is considered in the optimization. Details of the fitting processes are given in [15,19].

2.3 Subregions. The equation set consists of backward equations $p(h,s)$ for region 3. Region 3 is defined by

$$623.15\ \text{K}\leq T\leq 863.15\ \text{K}\quad\text{and}\quad p_{B23}^{97}(T)\leq p\leq 100\ \text{MPa}$$

where p_{B23}^{97} represents the B23-equation of IAPWS-IF97. The division of region 3 into subregions for the backward equations $p_3(h,s)$ is identical to the division for the previous region 3 backward equations [5,6] (see Fig. 2).

Table 1 shows the decision path to find the correct subregion for the function $p(h,s)$. For pressures less than the critical pressure p_c=22.064 MPa [20], the saturation line is the boundary between subregions 3a and 3b. If the specific enthalpy h is less than or equal to $h'_{3a}(s)$, Eq. (5), calculated from the entropy s on the saturated-liquid line, then the point to be calculated is located in subregion 3a. If h is greater than or equal to $h''_{2c3b}(s)$, Eq. (7), calculated on the saturated-vapor line, then the point is located in subregion 3b. Otherwise, the point is in the two-phase region. In this case, the saturation-pressure equation $p_{sat}^{97}(T)$ and the basic equation $f_3^{97}(v,T)$ of IAPWS-IF97 can be used to calculate the pressure from the given enthalpy and entropy.

For pressures greater than or equal to p_c, the boundary between subregions 3a and 3b corresponds to the critical isentrope $s=s_c$ (see Fig. 2). For the function $p(h,s)$, input points can be tested directly to identify the subregion, since the specific entropy is an independent variable. If the specific entropy s is less than or equal to

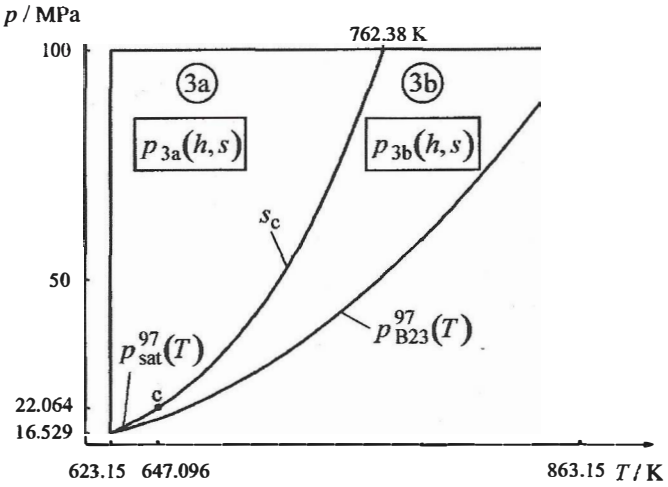

Fig. 2 Division of region 3 into two subregions 3a and 3b for the backward equations $p(h,s)$

$$s_c=4.412\ 021\ 482\ 234\ 76\ \text{kJ kg}^{-1}\ \text{K}^{-1}$$

then the point is located in subregion 3a; otherwise it is in subregion 3b. The critical entropy s_c is given with 15 digits to avoid numerical problems.

2.4 Backward Function $p(h,s)$

2.4.1 The Equations $p(h,s)$. The backward equation $p_{3a}(h,s)$ for subregion 3a has the dimensionless form

$$\frac{p_{3a}(h,s)}{p^*}=\pi(\eta,\sigma)=\sum_{i=1}^{33} n_i(\eta-1.01)^{I_i}(\sigma-0.750)^{J_i} \tag{2}$$

where $\pi=p/p^*$, $\eta=h/h^*$, and $\sigma=s/s^*$, with p^*=99 MPa, h^* =2300 kJ kg^{-1}, and s^*=4.4 kJ kg^{-1} K^{-1}. The coefficients n_i and exponents I_i and J_i of Eq. (2) are listed in Table 2.

$p_{3b}(h,s)$ for subregion 3b has the dimensionless form

$$\frac{p_{3b}(h,s)}{p^*}=\pi(\eta,\sigma)=\left[\sum_{i=1}^{35} n_i(\eta-0.681)^{I_i}(\sigma-0.792)^{J_i}\right]^{-1} \tag{3}$$

where $\pi=p/p^*$, $\eta=h/h^*$, and $\sigma=s/s^*$ with p^*=16.6 MPa, h^* =2800 kJ kg^{-1}, and s^*=5.3 kJ kg^{-1} K^{-1}. The coefficients n_i and exponents I_i and J_i of Eq. (3) are listed in Table 3.

To assist in computer-program verification of Eqs. (2) and (3), Table 4 contains test values for calculated pressures.

2.4.2 Consistency with IAPWS IF97. The maximum and root-mean-square relative differences between the calculated pressure (Eqs. (2) and (3)) and the IAPWS-IF97 basic equation $f_3^{97}(v,T)$, together with the permissible differences, are listed in Table 5. The maximum deviations are less than the permissible value. The critical pressure is met exactly by $p_3(h,s)$.

2.4.3 Consistency at the Subregion Boundary. The maximum relative difference between the backward equations $p_{3a}(h,s)$, Eq. (2), and $p_{3b}(h,s)$, Eq. (3), along the boundary s_c, is 0.00074%,

Table 1 Criteria for finding the subregion, 3a or 3b, for the backward function $p(h,s)$

	Subregion	
	3a	3b
For $p<p_c$	$h\leq h'_{3a}(s)$	$h\geq h''_{2c3b}(s)$
For $p\geq p_c$	$s\leq s_c$	$s>s_c$