# The International Association for the Properties of Water and Steam

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## Advisory Note No. 5: Industrial Calculation of the Thermodynamic Properties of Seawater

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The calculation procedure for the thermodynamic properties of seawater for industrial use provided in this Advisory Note is based on the IAPWS Formulation 2008 for the Thermodynamic Properties of Seawater [2]. For calculating the pure water contribution, the IAPWS Formulation 1995 for the Thermodynamic Properties of Ordinary Water Substance for General and Scientific Use [5] is replaced here by the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam [7]. For seawater in contact with ice, the Revised IAPWS Release on an Equation of State 2006 for H<sub>2</sub>O Ice Ih [11] is used. Further details about the formulation can be found in the article by H.-J. Kretzschmar *et al.* [1].

Further information about this Advisory Note and other documents issued by IAPWS can be obtained from the Executive Secretary of IAPWS or from http://www.iapws.org.

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# 1. Nomenclature

Symbol	Physical quantity
b	Molality of seawater
$c_p$	Specific isobaric heat capacity of seawater
C <sub>V</sub>	Specific isochoric heat capacity of seawater
f	Specific Helmholtz free energy of seawater
8	Specific Gibbs free energy of seawater
$g^{\mathrm{BI}}$	Specific Gibbs free energy of brine-ice mixture (sea ice)
$g^{\rm BV}$	Specific Gibbs free energy of brine-vapor mixture
$g^{\mathrm{Ih}}$	Specific Gibbs free energy of ice Ih
$g^{S}$	Saline part of the specific Gibbs free energy of seawater
$g^{\mathrm{vap}}$	Specific Gibbs free energy of water vapor
$g^{W}$	Water part of the specific Gibbs free energy of seawater
h	Specific enthalpy of seawater
$h^{W}$	Specific enthalpy of water
т	Mass of seawater including water vapor or ice
m <sub>S</sub>	Mass of sea salt in seawater
M <sub>S</sub>	Molar mass of sea salt
р	Pressure
$p_{b}$	Boiling pressure of seawater
$p_{\mathrm{f}}$	Freezing pressure of seawater
<i>p</i> <sub>osm</sub>	Osmotic pressure of seawater
$p_{\rm t}$	Triple-point pressure of seawater
$R_{ m W}$	Specific gas constant of water
<i>R</i> <sub>m</sub>	Molar gas constant
Re	Real part of a complex number
RMS	Root-mean-square value of a quantity
S	Salinity, mass fraction of sea salt in seawater including water vapor or ice
S <sub>b</sub>	Salinity of boiling brine
$S_{\rm f}$	Salinity of freezing brine
S <sub>n</sub>	Normal salinity of seawater
S	Specific entropy of seawater
$s^{W}$	Specific entropy of water
Т	Absolute temperature (ITS-90)
T <sub>b</sub>	Boiling temperature of seawater

Symbol	Physical quantity
$T_{\rm f}$	Freezing temperature of seawater
$T_{\rm t}$	Triple-point temperature of seawater
и	Uncertainty of a property given in Table 3
и	Specific internal energy of seawater
$u^{W}$	Specific internal energy of water
v	Specific volume of seawater
W	Speed of sound of seawater
x	Mass fraction of the vapor phase in the brine-vapor mixture (vapor fraction)
у	Mass fraction of the ice phase in the brine-ice mixture (ice fraction)
$\alpha_v$	Isobaric cubic expansion coefficient (thermal expansion coefficient) of seawater
β	Haline contraction coefficient of seawater
γ	Reduced Gibbs free energy
к	Isentropic exponent of seawater
κ <sub>T</sub>	Isothermal compressibility of seawater
μ	Relative chemical potential of seawater
$\mu_{ m S}$	Chemical potential of sea salt in seawater
$\mu_{ m W}$	Chemical potential of water in seawater
ξ	Reduced salinity root
$\pi$	Reduced pressure
ρ	Mass density of seawater
τ	Reduced or inverse reduced temperature
$\phi$	Osmotic coefficient of seawater

#### 2. Introductory Remarks

In 2008, IAPWS adopted the IAPWS Formulation 2008 for the Thermodynamic Properties of Seawater [2, 3, 4], where the thermodynamic properties of seawater are calculated from an equation of state, consisting of a water part and a saline part. The water part is computed from the Helmholtz free energy equation of the IAPWS Formulation 1995 for the Thermodynamic Properties of Ordinary Water Substance for General and Scientific Use (IAPWS-95) [5, 6]. The saline part is formulated as a Gibbs free energy equation.

However, the iterative calculation of required properties from the IAPWS-95 Helmholtz free energy equation is computationally intensive, making its use less desirable in applications where computational speed is important. In modeling industrial desalination and cooling processes, it is more reasonable to use the Gibbs free energy equation of Region 1 of the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam (IAPWS-IF97) [7, 8] for the water part of the seawater formulation instead of IAPWS-95. In addition, the IAPWS-IF97 Industrial Formulation is used by industry for calculating properties of pure steam and water for power plants and their components.

The purpose of this Advisory Note is to document the use of IAPWS-IF97 instead of IAPWS-95 for calculating thermodynamic properties of the water part of the seawater formulation, to enable more efficient calculations. This industrial calculation of the thermodynamic properties of seawater will be called the industrial formulation in this document.

It should be noted that this usage differs from use of the simplified water Gibbs energy formulation adopted as the IAPWS Supplementary Release on a Computationally Efficient Thermodynamic Formulation for Liquid Water for Oceanographic Use [3, 9]. Reference [9] documents a formulation for liquid water thermodynamics to be used specifically for computational efficiency in oceanographic calculations, while the present Advisory Note describes the application of the existing IAPWS "industrial" formulation to calculate the water contribution to seawater properties in industrial settings where the use of IAPWS-IF97 is desirable for other reasons.

In this Advisory Note, the terms "seawater" and "brine" refer to a solution of sea salt in liquid water, the term "sea salt" refers to a mixed solute with the chemical Reference Composition described in [10], "brine-vapor mixture" is a mixture of brine with water vapor, and "brine-ice mixture" is a mixture of brine with ice Ih. For water vapor, the IAPWS Revised Release on the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam (IAPWS-IF97) [7, 8] is used and for ice Ih, the IAPWS Revised Release on the Equation of State 2006 for  $H_2O$  Ice Ih (IAPWS-2006) [11, 12] is used. Note that salinity *S* here refers to the so-called Absolute Salinity of seawater which differs from the Practical Salinity that is obtained from common salinometers or oceanographic devices [2, 4, 10, 13].

The IAPWS Releases [2, 7, 11] referred to in this Advisory Note are available at the IAPWS website http://www.iapws.org. All details of the equations and algorithms should be taken from those Releases.

#### 3. Thermodynamic Properties of Seawater

#### 3.1 The Fundamental Equation of State for Seawater

The IAPWS equation of state for the liquid mixture "seawater" [2, 3, 4] is in the form of the specific Gibbs free energy as a function of pressure *p*, temperature *T*, and salinity *S*, and reads

$$g(p,T,S) = g^{W}(p,T) + g^{S}(p,T,S).$$
<sup>(1)</sup>

As described in the following sections, the water part  $g^{W}(p,T)$  is computed from the industrial formulation IAPWS-IF97, and the saline part  $g^{S}(p,T,S)$  from the IAPWS seawater formulation. The temperature *T* is based on the International Temperature Scale ITS-90. Salinity *S* is the mass fraction of sea salt in seawater

$$S = \frac{m_{\rm S}}{m} , \qquad (2)$$

where  $m_S$  is the mass of sea salt and m is the mass of the mixture seawater. The composition of sea salt is assumed to be the Reference Composition of Standard Seawater [10]. For pure water with S = 0, the saline part vanishes,  $g^{S}(p,T,0) = 0$ .

Using salinity *S*, the molality of seawater, defined as the quotient of moles of the solute sea salt per mass of the solvent water, is computed as

$$b = \frac{S}{M_{\rm S} \left(1 - S\right)},\tag{3}$$

where  $M_{\rm S}$  is the molar mass of sea salt. Its value can be taken from Table 1 of [2].

All thermodynamic properties and derivatives can be calculated from Eq. (1) by using the appropriate combinations of the Gibbs free energy equation and its derivatives with respect to p, T, and S. Relations between relevant thermodynamic properties and g(p,T,S), Eq. (1), and its derivatives are summarized in Table 1.<sup>1</sup>

In addition, any thermodynamic derivative of the variables p, T, v, s, u, h, f, and g, for example,  $(\partial h / \partial p)_{v,S}$ , can be determined from algebraic combinations of v, s,  $c_p$ ,  $\alpha_v$ , and  $\kappa_T$  as described in detail in IAPWS Advisory Note No. 3 [14].

<sup>&</sup>lt;sup>1</sup> Table 1 of [2] contains the value of the molar gas constant  $R_{\rm m}$  used in the 2008 IAPWS formulation for seawater.

Property	Relation
Specific volume v	$v(p,T,S) = g_p$
Density $\rho = \frac{1}{v}$	$\rho(p,T,S) = \frac{1}{g_p}$
Specific internal energy <i>u</i>	$u(p,T,S) = g - p g_p - T g_T$
Specific enthalpy $h = u + p v$	$h(p,T,S) = g - T g_T$
Specific entropy s	$s(p,T,S) = -g_T$
Specific Helmholtz free energy $f = u - T s$	$f(p,T,S) = g - p g_p$
Specific isobaric heat capacity $c_p = \left(\frac{\partial h}{\partial T}\right)_{p,S}$	$c_p(p,T,S) = -T g_{TT}$
Specific isochoric heat capacity $c_v = \left(\frac{\partial u}{\partial T}\right)_{v,S}$	$c_{v}\left(p,T,S\right) = T\left(\frac{g_{pT}^{2}}{g_{pp}} - g_{TT}\right)$
Cubic isobaric expansion coefficient $\alpha_v = \frac{1}{v} \left( \frac{\partial v}{\partial T} \right)_{p,S}$	$\alpha_{v}(p,T,S) = \frac{g_{pT}}{g_{p}}$
Isothermal compressibility $\kappa_T = -\frac{1}{v} \left( \frac{\partial v}{\partial p} \right)_{T,S}$	$\kappa_T(p,T,S) = -\frac{g_{pp}}{g_p}$
Speed of sound $w = v \sqrt{-\left(\frac{\partial p}{\partial v}\right)}_{s,S}$	$w(p,T,S) = g_p \sqrt{\frac{g_{TT}}{\left(g_{pT}^2 - g_{pp} g_{TT}\right)}}$
Isentropic exponent $\kappa = -\frac{v}{p} \left(\frac{\partial p}{\partial v}\right)_{s,S}$	$\kappa(p,T,S) = \frac{1}{p} \frac{g_p g_{TT}}{\left(g_{pT}^2 - g_{pp} g_{TT}\right)}$
Relative chemical potential $\mu = \left(\frac{\partial g}{\partial S}\right)_{p,T}$	$\mu(p,T,S) = g_S$
Chemical potential of water $\mu_{\rm W} = g - S \mu$	$\mu_{\rm W}(p,T,S) = g - S g_S$
Chemical potential of sea salt $\mu_{\rm S} = \mu + \mu_{\rm W}$	$\mu_{\mathrm{S}}(p,T,S) = g + (1-S)g_{S}$
Osmotic coefficient $\phi = \frac{\left(g^{W} - \mu_{W}\right)}{b R_{m} T}$	$\phi(p,T,S) = -\frac{\left(g^{S} - Sg_{S}\right)}{bR_{m}T}$
Haline contraction coefficient $\beta = \frac{1}{\rho} \left( \frac{\partial \rho}{\partial S} \right)_{p,T}$	$\beta(p,T,S) = -\frac{g_{pS}}{g_p}$

**Table 1** Relations between the relevant thermodynamic properties of seawater and g(p,T,S), Eq. (1), and its derivatives<sup>a</sup>

<sup>a</sup> Definitions of derivatives used in Table 1:

$$\begin{split} g_{p} &= \left(\frac{\partial g}{\partial p}\right)_{T,S} = \left(\frac{\partial g^{W}}{\partial p}\right)_{T} + \left(\frac{\partial g^{S}}{\partial p}\right)_{T,S}, \ g_{pp} = \left(\frac{\partial^{2} g}{\partial p^{2}}\right)_{T,S} = \left(\frac{\partial^{2} g^{W}}{\partial p^{2}}\right)_{T} + \left(\frac{\partial^{2} g^{S}}{\partial p^{2}}\right)_{T,S}, \\ g_{T} &= \left(\frac{\partial g}{\partial T}\right)_{p,S} = \left(\frac{\partial g^{W}}{\partial T}\right)_{p} + \left(\frac{\partial g^{S}}{\partial T}\right)_{p,S}, \ g_{TT} = \left(\frac{\partial^{2} g}{\partial T^{2}}\right)_{p,S} = \left(\frac{\partial^{2} g^{W}}{\partial T^{2}}\right)_{p} + \left(\frac{\partial^{2} g^{S}}{\partial T^{2}}\right)_{p,S}, \\ g_{pT} &= \left(\frac{\partial^{2} g}{\partial p \partial T}\right)_{S} = \left(\frac{\partial^{2} g^{W}}{\partial p \partial T}\right) + \left(\frac{\partial^{2} g^{S}}{\partial p \partial T}\right)_{S}, \ g_{S} = \left(\frac{\partial g}{\partial S}\right)_{p,T} = \left(\frac{\partial g^{S}}{\partial S}\right)_{p,T}, \\ g_{pS} &= \left(\frac{\partial^{2} g}{\partial p \partial S}\right)_{T} = \left(\frac{\partial^{2} g^{S}}{\partial p \partial S}\right)_{T} \end{split}$$

Backward functions are calculated by iteration from equations listed in Table 1. For example, for given pressure p, specific enthalpy h, and salinity S, the temperature T is calculated from the equation for h(p,T,S) by iteration.

#### 3.2 Water Part

The water part of Eq. (1) is calculated from the specific Gibbs free energy equation of IAPWS-IF97 region 1

$$\frac{g^{W}(p,T)}{R_{W}T} = \gamma(\pi,\tau) = \sum_{i=1}^{34} n_{i} \left(7.1 - \pi\right)^{I_{i}} \left(\tau - 1.222\right)^{J_{i}},\tag{4}$$

where the reduced Gibbs free energy is  $\gamma = g/(R_W T)$ , the reduced pressure is  $\pi = p/p^*$  and the inverse reduced temperature is  $\tau = T^*/T$ . The specific gas constant  $R_W$  of water, the reducing parameters  $p^*$  and  $T^*$ , the coefficients  $n_i$  and the exponents  $I_i$  and  $J_i$  are given in Section 5.1 of the IAPWS-IF97 Release [7].

Table 2 contains the relations for determining  $g^{W}(p,T)$  and its pressure and temperature derivatives from the reduced Gibbs free energy equation  $\gamma(\pi,\tau)$  and its derivatives. The equations for the derivatives of  $\gamma(\pi,\tau)$  can be taken from Table 4 in [7]. Using the equations of Table 2, the water parts in the equations of Table 1 can be determined.

Note that for temperatures less than the melting temperature of pure water [15, 16], or greater than the saturation temperature of pure water [6, 7], the IAPWS-IF97 Gibbs free energy equation of liquid water region 1, Eq. (4), is evaluated at conditions where the liquid phase of pure water is metastable. Investigations have shown that Eq. (4) can be reasonably extrapolated into this metastable region and even below 273.15 K, the minimum temperature of IAPWS-IF97. The accuracy of the thermodynamic properties of seawater calculated from IAPWS-IF97 is sufficient for industrial calculations. Section 7 contains a description of the deviations of IAPWS-IF97 from IAPWS-95.

**Table 2** Relations for determining  $g^{W}(p,T)$  and pressure and temperature derivatives from the reduced Gibbs free energy equation  $\gamma(\pi,\tau)$  and its derivatives of IAPWS-IF97 region  $1^{a}$ 

$$g^{W} = R_{W} T \gamma, \quad \left(\frac{\partial g^{W}}{\partial p}\right)_{T} = \frac{R_{W} T}{p} \pi \gamma_{\pi}, \quad \left(\frac{\partial^{2} g^{W}}{\partial p^{2}}\right)_{T} = \frac{R_{W} T}{p^{2}} \pi^{2} \gamma_{\pi\pi},$$

$$\left(\frac{\partial g^{W}}{\partial T}\right)_{p} = R_{W} \left(\gamma - \tau \gamma_{\tau}\right), \quad \left(\frac{\partial^{2} g^{W}}{\partial T^{2}}\right)_{p} = \frac{R_{W}}{T} \tau^{2} \gamma_{\tau\tau}, \quad \left(\frac{\partial^{2} g^{W}}{\partial p \partial T}\right) = \frac{R_{W} \pi}{p} (\gamma_{\pi} - \tau \gamma_{\pi\tau})$$

$$^{a} \gamma_{\pi} = \left(\frac{\partial \gamma}{\partial \pi}\right)_{\tau}, \quad \gamma_{\pi\pi} = \left(\frac{\partial^{2} \gamma}{\partial \pi^{2}}\right)_{\tau}, \quad \gamma_{\tau} = \left(\frac{\partial \gamma}{\partial \tau}\right)_{\pi}, \quad \gamma_{\tau\tau} = \left(\frac{\partial^{2} \gamma}{\partial \tau^{2}}\right)_{\pi}, \quad \gamma_{\pi\tau} = \left(\frac{\partial^{2} \gamma}{\partial \pi \partial \tau}\right)$$

### **3.3 Saline Part**

The saline part of Eq. (1) is calculated from the Gibbs free energy equation of the IAPWS formulation for seawater properties [2]

$$\frac{g^{\rm S}(p,T,S)}{g^{\,*}} = \sum_{k=0}^{5} \sum_{j=0}^{6} \left( g_{1jk} \xi^2 \ln \xi + \sum_{i=2}^{7} g_{ijk} \xi^i \right) \tau^j \pi^k \,, \tag{5}$$

where the reduced pressure is  $\pi = (p - p_0)/p^*$ , the reduced temperature is  $\tau = (T - T_0)/T^*$ , and the square root of the reduced salinity is  $\xi = \sqrt{S/S^*}$ . The constants  $g^*$ ,  $T^*$ ,  $p^*$ ,  $S^*$ ,  $T_0$ , and  $p_0$  are given in Table 1 of the IAPWS Seawater Release [2], and the coefficients  $g_{ijk}$  are given in Table 2 of [2].

Table 6 of [2] contains the relations for determining the following derivatives

$$\left(\frac{\partial g^{S}}{\partial p}\right)_{T,S} = g_{p}^{S}, \quad \left(\frac{\partial^{2} g^{S}}{\partial p^{2}}\right)_{T,S} = g_{pp}^{S}, \quad \left(\frac{\partial g^{S}}{\partial T}\right)_{p,S} = g_{T}^{S}, \quad \left(\frac{\partial^{2} g^{S}}{\partial T^{2}}\right)_{p,S} = g_{TT}^{S}, \quad \left(\frac{\partial^{2} g^{S}}{\partial p \partial T}\right)_{S} = g_{pT}^{S},$$

$$\left(\frac{\partial g^{S}}{\partial S}\right)_{p,T} = g_{S}^{S}, \quad \left(\frac{\partial^{2} g^{S}}{\partial p \partial S}\right)_{T} = g_{pS}^{S}$$

from Eq. (5) which are used in Table 1.

#### 4. Colligative Properties

#### 4.1 Phase Equilibrium between Seawater and Water Vapor

For computation of the equilibrium between seawater and water vapor, the required thermodynamic condition is the equality of the chemical potential of H<sub>2</sub>O in seawater  $\mu_W$  with the specific Gibbs energy of water vapor  $g^{\text{vap}}$ . Here, chemical potentials are expressed on a mass basis, which is the usual (molar) chemical potential divided by molar mass.

For the phase equilibrium between seawater and water vapor, the following condition must be fulfilled:

$$\mu_{\mathrm{W}}(p,T,S) = g^{\mathrm{vap}}(p,T), \tag{6}$$

which can be written equivalently in terms of the osmotic coefficient  $\phi$  as

$$bR_{\rm m}T\phi(p,T,S) = g^{\rm W}(p,T) - g^{\rm vap}(p,T), \qquad (7)$$

where  $\mu_W(p,T,S)$  and  $\phi(p,T,S)$  are from Table 1, and  $g^W(p,T)$  is from Eq. (4). The molality *b* is determined from Eq. (3) and the value of the molar gas constant  $R_m$  is taken from Table 1 of [2]. The Gibbs free energy of water vapor  $g^{vap}(p,T)$  is calculated from the Gibbs free energy equation of the IAPWS-IF97 region 2,

$$\frac{g^{\mathrm{vap}}(p,T)}{R_{\mathrm{W}}T} = \gamma(\pi,\tau) = \gamma^{\mathrm{o}}(\pi,\tau) + \gamma^{\mathrm{r}}(\pi,\tau), \qquad (8)$$

with the ideal gas part,

$$\gamma^{0} = \ln \pi + \sum_{i=1}^{9} n_{i}^{0} \tau^{J_{i}^{0}} , \qquad (9)$$

and the residual part,

$$\gamma^{r} = \sum_{i=1}^{43} n_{i} \pi^{I_{i}} \left(\tau - 0.5\right)^{J_{i}}.$$
(10)

In Eqs. (8) to (10),  $\gamma = g^{\text{vap}} / (R_{\text{W}}T)$  is the reduced Gibbs free energy,  $\pi = p/p^*$  is the reduced pressure, and  $\tau = T^*/T$  is the inverse reduced temperature. The specific gas constant  $R_{\text{W}}$  of water, the reducing parameters  $p^*$  and  $T^*$ , the coefficients  $n_i^0$  and  $n_i$ , and the exponents  $I_i$ ,  $J_i^0$ , and  $J_i$  are given in Section 6.1 of the IAPWS-IF97 Release [7].

Using Eq. (6) or (7), the boiling temperature  $T = T_b$  can be calculated by iteration from pressure *p* and salinity *S*, or boiling pressure  $p = p_b$  from temperature *T* and salinity *S*, or brine salinity  $S = S_b$  from *p* and *T*. At a given equilibrium state between seawater and water vapor, the properties of the seawater (brine) phase are calculated from Eq. (1) and the properties of the water vapor result from Eq. (8), see Section 4.5.

At brine-vapor equilibrium, vapor is superheated, i.e., at given pressure, the temperature is higher than the saturation temperature of pure water or the pressure at given temperature is below the saturation pressure of pure water.

Note that the Gibbs free energy of liquid water,  $g^{W}(p,T)$  in Eq. (7), is evaluated from IAPWS-IF97 at conditions where the liquid phase of pure water is metastable. Due to salinity, the boiling temperature elevation can be up to 2 K.

Table A2 shows selected boiling temperatures for given pressures and salinities.

#### 4.2 Phase Equilibrium between Seawater and Ice

For the phase equilibrium between seawater and ice Ih, the following condition has to be fulfilled:

$$\mu_{\mathrm{W}}(p,T,S) = g^{\mathrm{Ih}}(p,T), \qquad (11)$$

or equivalently:

$$bR_{\rm m}T\phi(p,T,S) = g^{\rm W}(p,T) - g^{\rm lh}(p,T), \qquad (12)$$

with  $\mu_W(p,T,S)$  and  $\phi(p,T,S)$  from Table 1 and  $g^W(p,T)$  from Eq. (4). The molality *b* is determined from Eq. (3) and the value of the molar gas constant  $R_m$  is taken from Table 1 of [2]. The Gibbs free energy of ice Ih  $g^{\text{Ih}}(p,T)$  is calculated from the Gibbs free energy equation of the corresponding IAPWS Release [11]:

$$g^{\ln}(p,T) = g_0(p) - s_0 T_t^W \tau + T_t^W \operatorname{Re}\left\{\sum_{k=1}^2 r_k \left[ (t_k - \tau) \ln(t_k - \tau) + (t_k + \tau) \ln(t_k + \tau) - 2t_k \ln t_k - \frac{\tau^2}{t_k} \right] \right\}.$$
 (13)

The functions  $g_0(p)$  and  $r_k(p)$ , the triple-point temperature of water  $T_t^W$ , and the reduced temperature  $\tau$  are given in [11]. The real constant  $s_0$  as well as the complex constants  $t_1$ ,  $r_1$ , and  $t_2$  are listed in Table 2 of [11].

Using Eq. (11) or (12), the freezing temperature  $T = T_f$  can be calculated by iteration from pressure *p* and salinity *S*, or the freezing pressure  $p = p_f$  from *T* and *S*, or brine salinity  $S = S_f$  from *p* and *T*. At a given equilibrium state between seawater and ice, the properties of the seawater (brine) phase are calculated from Eq. (1) and the properties of the ice phase result from Eq. (13), see Section 4.6.

At brine-ice equilibrium, the temperature is lower than the melting temperature at a given pressure of pure ice or the pressure at a given temperature is below the melting pressure of pure ice.

Note that the Gibbs free energy of liquid water  $g^{W}(p,T)$  in Eq. (12) is evaluated from IAPWS-IF97 at conditions where the liquid phase of pure water is metastable. Due to salinity, the freezing-point depression can be up to 8 K.

Table A3 shows selected freezing temperatures for given pressures and salinities.

### 4.3 Triple-Point Temperatures and Pressures

The triple-point temperatures  $T = T_t$  and triple-point pressures  $p = p_t$  of seawater are calculated for given salinity *S* by iteration from both Eqs. (6) and (11) or from both Eqs. (7) and (12).

Table A4 contains selected triple-point temperatures and triple-point pressures for given salinities.

#### **4.4 Osmotic Pressure**

On the two sides of a membrane permeable to water but not to sea salt, equilibrium between liquid water and seawater causes an excess pressure of seawater, the osmotic pressure  $p_{osm}$ , computed by iteration from the condition

$$\mu_{\mathrm{W}}\left[\left(p+p_{\mathrm{osm}}\right),T,S\right] = g^{\mathrm{W}}\left(p,T\right),\tag{14}$$

or equivalently from

$$b R_{\rm m} T \phi \left[ \left( p + p_{\rm osm} \right), T, S \right] = g^{\rm W} \left[ \left( p + p_{\rm osm} \right), T \right] - g^{\rm W} \left( p, T \right) , \qquad (15)$$

with  $\mu_{\rm W}$  and  $\phi$  from Table 1 and  $g^{\rm W}$  from Eq. (4). The molality *b* is determined from Eq. (3) and the value of the molar gas constant  $R_{\rm m}$  is taken from Table 1 of [2].

Using Eq. (14) or (15), the osmotic pressure  $p_{osm}$  is calculated by iteration for pressure p, temperature T, and salinity S.

#### 4.5 Properties of Brine-Vapor Mixture

The properties for the mixture of brine and water vapor, termed brine-vapor mixture, can be computed from the combined Gibbs free energy equation

$$g^{\rm BV}(p,T,S) = (1-x)g(p,T,S_{\rm b}) + xg^{\rm vap}(p,T),$$
(16)

where *p* is the given pressure, *T* is the given temperature, and *S* is the given salinity of the seawater mixture including water vapor. The boiling-brine salinity  $S_b = S_b(p,T)$  is calculated for pressure *p* and temperature *T* using Eq. (6) by iteration. The mass fraction of water vapor in the seawater mixture (vapor fraction) *x* can be determined from the equation

$$x = 1 - \frac{S}{S_{\rm b}(p,T)} \,. \tag{17}$$

The Gibbs free energy of the brine  $g(p,T,S_b)$  is calculated from Eq. (1) and the Gibbs free energy of water vapor  $g^{\text{vap}}(p,T)$  from Eq. (8).

Brine-vapor properties can be computed from the equation  $g^{\text{BV}}(p,T,S)$ , Eq. (16), and its derivatives in analogy to Table 1.<sup>2</sup> The derivatives of the included equation  $g^{\text{vap}}(p,T)$ , Eq. (8),

<sup>&</sup>lt;sup>2</sup> The partial derivatives of  $g^{\text{BV}}$  with respect to *p* or *T* include differentiation of  $S_b(p,T)$  via the chain rule, making use of Eq. (6). The related terms vanish identically for the first derivatives  $(g_p^{\text{BV}}, g_T^{\text{BV}})$ . This is true, e.g., for the properties *v*, *h*, *u*, and *s*. But they provide the dominant phase-change contributions such as latent heat to the second derivatives  $(g_{pp}^{\text{BV}}, g_{TT}^{\text{BV}}, g_{pT}^{\text{BV}})$  [17, 18] which are used, e.g., for the cubic isobaric expansion coefficient  $\alpha_v$ .

for water vapor can be taken from [7]. As an example, the specific enthalpy of brine-vapor mixture is calculated from the equation

$$h^{\mathrm{BV}}(p,T,S) = (1-x)h(p,T,S_{\mathrm{b}}) + xh^{\mathrm{vap}}(p,T)$$
(18)

where the determination of  $h(p,T,S_b)$  is given in Table 1 and of  $h^{vap}(p,T)$  in [7].

Further properties such as specific volume  $v^{BV}(p,T,S)$ , specific internal energy  $u^{BV}(p,T,S)$ , and specific entropy  $s^{BV}(p,T,S)$  are computed analogously.

#### 4.6 Properties of Brine-Ice Mixture

The properties for the mixture of brine and ice Ih, termed brine-ice mixture or sea ice, can be computed from the combined Gibbs free energy equation

$$g^{\rm BI}(p,T,S) = (1-y)g(p,T,S_{\rm f}) + yg^{\rm Ih}(p,T),$$
(19)

where *p* is the given pressure, *T* is the given temperature, and *S* is the given salinity of the seawater mixture including ice Ih. The freezing-brine salinity  $S_f = S_f(p,T)$  is calculated for pressure *p* and temperature *T* using Eq. (11) by iteration. The mass fraction of ice Ih in the seawater mixture (ice fraction) *y* can be determined from the equation

$$y = 1 - \frac{S}{S_{\rm f}(p,T)}$$
 (20)

The Gibbs free energy of the brine  $g(p,T,S_f)$  is calculated from Eq. (1) and the Gibbs free energy of ice Ih  $g^{\text{Ih}}(p,T)$  from Eq. (13).

Brine-ice mixture properties can be computed from the equation  $g^{BI}(p,T,S)$ , Eq. (19), and its derivatives in analogy to Table 1.<sup>3</sup> The derivatives of the included equation  $g^{Ih}(p,T)$ , Eq. (13), for ice Ih can be taken from [11]. As an example, the specific enthalpy of brine-ice mixture is calculated from the equation

$$h^{\rm BI}(p,T,S) = (1-y)h(p,T,S_{\rm f}) + yh^{\rm Ih}(p,T),$$
(21)

where the determination of  $h(p,T,S_f)$  is given in Table 1 and of  $h^{\text{Ih}}(p,T)$  in [11].

Further properties such as the specific volume  $v^{\text{BI}}(p,T,S)$ , specific internal energy  $u^{\text{BI}}(p,T,S)$ , and specific entropy  $s^{\text{BI}}(p,T,S)$  are computed analogously.

<sup>&</sup>lt;sup>3</sup> The partial derivatives of  $g^{\text{BI}}$  with respect to *p* or *T* include differentiation of  $S_f(p,T)$  via the chain rule, making use of Eq. (11). The related terms vanish identically for the first derivatives  $(g_p^{\text{BI}}, g_T^{\text{BI}})$ . This is true, e.g., for the properties *v*, *h*, *u*, and *s*. But they provide the dominant phase-change contributions such as latent heat to the second derivatives  $(g_{pp}^{\text{BI}}, g_{TT}^{\text{BI}}, g_{pT}^{\text{BI}})$  [17, 18] which are used, e.g., for the cubic isobaric expansion coefficient  $\alpha_v$ .

### 5. Reference States

According to the IAPWS Formulation 2008 for the Thermodynamic Properties of Seawater [2], the specific enthalpy and the specific entropy of seawater calculated from Eq. (1) have been set to zero at p = 0.101325 MPa, T = 273.15 K, and  $S = S_n = 0.03516504$  kg kg<sup>-1</sup>:

h = 0 and s = 0,

where  $S_n$  is the normal salinity of seawater [10].

The reference state of Eq. (1) for pure liquid water, water vapor, and ice corresponds to the following values. The specific internal energy and the specific entropy of saturated liquid water at the triple point (T = 273.16 K, p = 0.000611657 MPa, and S = 0) are:

u = 0 and s = 0.

From u = 0 it follows for the specific enthalpy

 $h = 0.000611783 \text{ kJ kg}^{-1}$ 

at this state point. These values correspond to the reference state of IAPWS-IF97.

### 6. Range of Validity

The equation of state, Eq. (1), is valid for Standard Seawater with sea salt of the Reference Composition [10] in certain regions inside the following pressure, temperature, and salinity ranges

 $0.3 \text{ kPa} \le p \le 100 \text{ MPa}$ ,  $261 \text{ K} \le T \le 353 \text{ K}$ , and  $0 \le S \le 0.12 \text{ kg kg}^{-1}$ .

All properties of Table 1 can be calculated with high accuracy in the temperature range

```
T_{\rm f}(p,S) \leq T \leq 313 \, {\rm K}
```

for 0.101325 MPa  $\leq p \leq 100$  MPa and  $0 \leq S \leq 0.042$  kg kg<sup>-1</sup> (Region A in [2])

or

for  $0.3 \text{ kPa} \le p \le 0.101325 \text{ MPa}$  and  $0 \le S \le 0.05 \text{ kg kg}^{-1}$  (Region B in [2]).

Outside the above two regions, properties of Table 1 except those depending on pressure derivatives can be calculated with high accuracy near standard atmospheric pressure (p = 0.101325 MPa).

for  $T_{\rm f}(p,S) \le T \le 353 \,\text{K}$  and  $0 \le S \le 0.12 \,\text{kg kg}^{-1}$  (Region C in [2]).

Density is very well extrapolated by this formulation up to the highest salinity values at low temperatures. In the range of higher salinity and higher temperature near standard atmospheric pressure, density is described with lower accuracy and quantities computed from its extrapolated derivatives may even be invalid. Due to the lack of experimental data, no statements could be made in the release on the IAPWS formulation 2008 [2] about the accuracy at high pressures for temperatures greater than 313 K and salinities greater than 0.042.

Restrictions regarding certain properties outside these ranges are described in detail in Section 6 of the IAPWS Release [2]. Reference [2] also contains a graphical representation of the range of validity in a *p*-*T*-*S* plot.

### 7. Uncertainty

A summary of estimated combined standard uncertainties u (coverage factor k=1) of selected quantities using IAPWS-95 for the water part in certain regions of the *p*-*T*-*S* space is given in Table 7 in the IAPWS release on the IAPWS formulation 2008 [2]. By using IAPWS-IF97 instead of IAPWS-95, some deviations caused by the difference between IAPWS-IF97 and IAPWS-95 must be added to the uncertainties. The total uncertainty using IAPWS-IF97 can be estimated by the equation

$$u = \sqrt{u_{08}^2 + \Delta_{\rm RMS}^2} , \qquad (22)$$

where  $u_{08}$  represents the uncertainty of the 2008 IAPWS seawater formulation [2] (which uses IAPWS-95 for pure-water properties), and  $\Delta_{\text{RMS}}$  represents the root-mean-square of the deviations caused by the difference between IAPWS-IF97 and IAPWS-95 and is computed from  $\Delta_{\text{RMS}} = \sqrt{\frac{1}{N} \sum_{n=1}^{N} (\Delta x_n)^2}$ , where  $\Delta x_n$  can be either absolute or percentage difference between the corresponding quantities *x*; *N* is the number of  $\Delta x_n$  values (100 000 points uniformly distributed

over the respective range of validity were used to calculate Table 3).

A summary of estimated values  $u_{08}$ ,  $\Delta_{\text{RMS}}$ , and u is given in Table 3.

#### 8. Computing Time for the IAPWS Seawater Functions

One important reason to use IAPWS-IF97 instead of IAPWS-95 for calculating the water part of the IAPWS seawater functions is the computing-speed difference between these two standards. The relations of the computing speed of the seawater functions of the 2008 IAPWS seawater formulation [2] (which uses IAPWS-95) in comparison with the industrial formulation for seawater given herein (using IAPWS-IF97) were investigated and reported in [1], where improvements in speed by factors on the order of 100 or 200 were reported. While the observed improvement will depend on the efficiency with which IAPWS-95 is programmed (and also on details of the machine and compiler used), the use of IAPWS-IF97 clearly produces a significant reduction in computing time.

	<i>S</i> interval	T interval	<i>p</i> interval		٨	
Quantity	kg kg <sup>-1</sup>	K	MPa	<i>u</i> <sub>08</sub>	$\Delta_{ m RMS}$	и
$\Delta  ho$	0-0.04	273–313	0.1	$4 \times 10^{-6}$	$2.9 \times 10^{-6}$	$5 \times 10^{-6}$
$\rho$	0.04-0.05	288–303	0.1	$1 \times 10^{-5}$	$1.3 \times 10^{-6}$	$1 \times 10^{-5}$
	0.005-0.04	273–313	0.1–10	$1 \times 10^{-5}$	$2.6 \times 10^{-6}$	$1 \times 10^{-5}$
	0.005-0.04	273–313	10-100	$2 \times 10^{-5}$	$5.3 \times 10^{-6}$	$2 \times 10^{-5}$
	0.04-0.12	293–313	0.1	$3 \times 10^{-4}$	$4.2 \times 10^{-6}$	$3 \times 10^{-4}$
	0-0.04	313–333	0.1	$4 \times 10^{-4}$	$1.3 \times 10^{-5}$	$4 \times 10^{-4}$
	0.04-0.08	313–333	0.1	$9 \times 10^{-4}$	$1.3 \times 10^{-5}$	$9 \times 10^{-4}$
	0-0.04	333–353	0.1	$1.4 \times 10^{-3}$	$1.5 \times 10^{-5}$	$1.4 \times 10^{-3}$
	0.08-0.12	313–333	0.1	$3 \times 10^{-3}$	$1.3 \times 10^{-5}$	$3 \times 10^{-3}$
	0.04-0.08	333–353	0.1	$4 \times 10^{-3}$	$1.5 \times 10^{-5}$	$4 \times 10^{-3}$
	0.08-0.12	333–353	0.1	$1.3 \times 10^{-2}$	$1.6 \times 10^{-5}$	$1.3 \times 10^{-2}$
$ \Delta lpha_v $	0.01-0.03	267–274	0.7–33	$6 \times 10^{-7}  \mathrm{K}^{-1}$	$1 \times 10^{-6}  \mathrm{K}^{-1}$	$1 \times 10^{-6}  \mathrm{K}^{-1}$
$\Delta w$	0.029–0.043	273–308	0.1–2	$3 \times 10^{-5}$	$8.2 \times 10^{-4}$	$8.2 \times 10^{-4}$
W	0.029–0.043	273-303	0.1–5	$3 \times 10^{-5}$	$6.4 \times 10^{-4}$	$6.4 \times 10^{-4}$
	0.033-0.037	273–278	0.1–100	$5 \times 10^{-5}$	$1.8 \times 10^{-4}$	$1.8 \times 10^{-4}$
$\Delta p^{\mathrm{vap}}$	0.02-0.12	293–353	0.002-0.05	$1 \times 10^{-3}$	$3.9 \times 10^{-5}$	$1 \times 10^{-3}$
$p^{\mathrm{vap}}$	0.018-0.04	298	0.003	$2 \times 10^{-4}$	$1.5 \times 10^{-5}$	$2 \times 10^{-4}$
$\left \Delta T_{\mathrm{f}}\right $	0.004-0.04	271–273	0.1	2 mK	0.014 mK	2 mK
$\left \Delta T_{\rm b}\right $	0.006–0.07	333–353	0.02–0.05	2 mK	1.2 mK	2.3 mK
$\Delta \phi$	0.004-0.04	273	0.1	$2 \times 10^{-3}$	_ a	$2 \times 10^{-3}$
$\phi$	0.0017-0.038	298	0.1	$2 \times 10^{-3}$	_ a	$2 \times 10^{-3}$
	0.01-0.12	273–278	0.1	$3 \times 10^{-3}$	_ a	$3 \times 10^{-3}$
$\Delta c_p^{\rm S}$	0-0.04	273–313	0.1	$0.5 \text{ J kg}^{-1} \text{ K}^{-1}$	_ a	$0.5 \text{ J kg}^{-1} \text{ K}^{-1}$
$\left \Delta c_{p}\right $	0-0.12	273–353	0.1	$4 J kg^{-1} K^{-1}$	$1.3 \text{ J kg}^{-1} \text{ K}^{-1}$	$4.2 \text{ J kg}^{-1} \text{ K}^{-1}$

**Table 3** Uncertainties of selected quantities in certain regions of the *p*-*T*-*S* space estimated for Eq. (1)

 $^{a}$  The quantity is only dependent on the saline part of Eq. (1).

### 9. Computer-Program Verification

To assist the user in computer-program verification, test values are given in Table A1. It contains values for the specific Gibbs energy, g(p,T,S), together with the corresponding derivatives and some thermodynamic properties. For an easy check of the Gibbs free energy functions of the water part  $g^{W}(p,T)$  and of the saline part  $g^{S}(p,T,S)$ , the results of both parts are reported separately in Table A1.

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## APPENDIX

**Table A1** Numerical check values for the water part computed from  $g^{W}(p,T)$ , Eq. (4), and its derivatives, for the saline part computed from  $g^{S}(p,T,S)$ , Eq. (5), and its derivatives, for the seawater properties computed from the Gibbs function g(p,T,S), Eq. (1) and its derivatives and for selected seawater properties of Table 1 at given points (p,T,S)

Properties at p = 0.101325 MPa, T = 273.15 K,  $S = S_n = 0.03516504 \text{ kg kg}^{-1}$ 

Quantity	Water part	Saline part	Property of seawater	Unit
g	$0.101\ 359\ 446 \times 10^3$	$-0.101\ 342\ 742 \times 10^3$	$0.167~05 \times 10^{-1}$	J kg <sup>-1</sup>
$(\partial g/\partial p)_{T,S}$	$0.100\ 015\ 572  imes 10^{-2}$	$-0.274~957~224\times 10^{-4}$	$0.972~659~998 \times 10^{-3}$	$m^3 kg^{-1}$
$(\partial^2 g/\partial p^2)_{T,S}$	$-0.508\ 885\ 499\times 10^{-12}$	$0.581\ 535\ 172\times 10^{-13}$	$-0.450\ 731\ 982\times 10^{-12}$	$m^3 kg^{-1} Pa^{-1}$
$(\partial g/\partial T)_{p,S}$	0.147 711 823	-0.147 643 376	$0.684\ 47  imes 10^{-4}$	$\mathbf{J} \ \mathbf{kg}^{-1} \ \mathbf{K}^{-1}$
$(\partial^2 g/\partial T^2)_{p,S}$	$-0.154\ 473\ 013\times 10^2$	0.852 861 151	$-0.145\ 944\ 401\times 10^2$	$\rm J~kg^{-1}~K^{-2}$
$(\partial^2 g/\partial p \ \partial T)_S$	$-0.676\ 992\ 620\times 10^{-7}$	$0.119\ 286\ 787\times 10^{-6}$	$0.515\ 875\ 254  imes 10^{-7}$	$m^3 kg^{-1} K^{-1}$
$(\partial g/\partial S)_{p,T}$	0	$0.639\ 974\ 067  imes 10^5$	$0.639\ 974\ 067  imes 10^5$	${ m J~kg}^{-1}$
$(\partial^2 g/\partial p \ \partial S)_T$	0	$-0.759\ 615\ 412\times 10^{-3}$	$-0.759\ 615\ 412\times 10^{-3}$	$m^3 kg^{-1}$
v	$0.100\ 015\ 572\  imes 10^{-2}$	$-0.274~957~224\times 10^{-4}$	$0.972~659~995 \times 10^{-3}$	$m^3 kg^{-1}$
и	$-0.403\ 288\ 161\times 10^{-1}$	$-0.582\ 279\ 494\times 10^{-1}$	$-0.985\ 567\ 655\times 10^{-1}$	kJ k $g^{-1}$
h	$0.610\ 119\ 617\times 10^{-1}$	$-0.610\ 139\ 535\times 10^{-1}$	$-0.199\ 16 \times 10^{-5}$	kJ k $g^{-1}$
S	$-0.147\ 711\ 823\times 10^{-3}$	$0.147~643~376 \times 10^{-3}$	$-0.684 47 \times 10^{-7}$	$kJ kg^{-1} K^{-1}$
$c_p$	$0.421~943~034 \times 10^{1}$	-0.232 959 023	$0.398~647~132 \times 10^{1}$	$kJ kg^{-1} K^{-1}$
W	$0.140\ 243\ 979  imes 10^4$	_ a	$0.144~907~123 \times 10^4$	${ m m~s}^{-1}$
$\mu_{ m W}$	0.101 359 446	$-0.235\ 181\ 411 \times 10^{1}$	$-0.225\ 045\ 466 \times 10^{1}$	kJ k $g^{-1}$

<sup>a</sup> This value cannot be computed from  $g^{S}$  alone because it is a nonlinear expression in g

# Table A1 – continued

Properties at p = 0.101325 MPa, T = 353 K,  $S = 0.1 \text{ kg kg}^{-1}$ 

Quantity	Water part	Saline part	Property of seawater	Unit
g	$-0.446\ 091\ 363\times 10^5$	$0.150\ 871\ 740 \times 10^5$	$-0.295\ 219\ 623 \times 10^5$	J kg <sup>-1</sup>
$(\partial g/\partial p)_{T,S}$	$0.102\ 891\ 627\times 10^{-2}$	$-0.579\ 227\ 286\times 10^{-4}$	$0.970~993~546 \times 10^{-3}$	$m^3 kg^{-1}$
$(\partial^2 g/\partial p^2)_{T,S}$	$-0.473\ 220\ 602\times 10^{-12}$	$0.213\ 086\ 154\times 10^{-12}$	$-0.260\ 134\ 448\times 10^{-12}$	$m^3 kg^{-1} Pa^{-1}$
$(\partial g/\partial T)_{p,S}$	$-0.107\;357\;342\times10^4$	$0.156\ 230\ 907 \times 10^3$	$-0.917\ 342\ 513\times 10^3$	$\rm J~kg^{-1}~K^{-1}$
$(\partial^2 g/\partial T^2)_{p,S}$	$-0.118\ 849\ 543\times 10^2$	$0.127\ 922\ 649  imes 10$	$-0.106\ 057\ 278 \times 10^2$	$J kg^{-1} K^{-2}$
$(\partial^2 g/\partial p \ \partial T)_S$	$0.659\ 344\ 070 \times 10^{-6}$	$0.803\ 061\ 596  imes 10^{-6}$	$0.146\ 240\ 567\times 10^{-5}$	$m^3 kg^{-1} K^{-1}$
$(\partial g/\partial S)_{p,T}$	0	$0.251\ 957\ 276 \times 10^6$	$0.251\ 957\ 276 \times 10^6$	$\mathrm{J}~\mathrm{kg}^{-1}$
$(\partial^2 g/\partial p \ \partial S)_T$	0	$-0.305\ 957\ 802\times 10^{-3}$	$-0.305\ 957\ 802 \times 10^{-3}$	$m^3 kg^{-1}$
v	$0.102\ 891\ 627  imes 10^{-2}$	$-0.579\ 227\ 286\times 10^{-4}$	$0.970\ 993\ 546  imes 10^{-3}$	$m^3 kg^{-1}$
и	$0.334\ 258\ 026 \times 10^3$	$-0.400\ 564\ 673\times 10^2$	$0.294\ 201\ 559 \times 10^3$	kJ k $g^{-1}$
h	$0.334\ 362\ 281 \times 10^3$	$-0.400\ 623\ 363\times 10^2$	$0.294\ 299\ 945 \times 10^3$	kJ k $g^{-1}$
S	$0.107\ 357\ 342 \times 10^1$	-0.156 230 907	0.917 342 513	$\rm kJ~kg^{-1}~K^{-1}$
$c_p$	$0.419\ 538\ 887  imes 10^1$	-0.451 566 952	$0.374\ 382\ 192 \times 10^1$	$\rm kJ~kg^{-1}~K^{-1}$
W	$0.155\ 710\ 273  imes 10^4$	_ <sup>a</sup>	$0.401\;505\;044 \times 10^4$	${ m m~s}^{-1}$
$\mu_{ m W}$	$-0.446\ 091\ 363\times 10^2$	$-0.101\ 085\ 536\times 10^2$	$-0.547\ 176\ 899 \times 10^2$	kJ kg <sup>-1</sup>

<sup>a</sup> This value cannot be computed from  $g^{S}$  alone because it is a nonlinear expression in g

# Table A1 – continued

Properties at p = 100 MPa, T = 273.15 K,  $S = S_n = 0.03516504 \text{ kg kg}^{-1}$ 

Quantity	Water part	Water part Saline part		Unit
g	$0.977\ 302\ 204 \times 10^5$	$-0.260\ 093\ 051\times 10^4$	$0.951\ 292\ 899 \times 10^5$	J kg <sup>-1</sup>
$(\partial g/\partial p)_{T,S}$	$0.956\ 686\ 939\times 10^{-3}$	$-0.229\ 123\ 842\times 10^{-4}$	$0.933\ 774\ 555  imes 10^{-3}$	$m^3 kg^{-1}$
$(\partial^2 g/\partial p^2)_{T,S}$	$-0.371\ 594\ 622\times 10^{-12}$	$0.357\;345\;736 \times 10^{-13}$	$-0.335\ 860\ 049\times 10^{-12}$	$m^3 kg^{-1} Pa^{-1}$
$(\partial g/\partial T)_{p,S}$	$0.858\ 228\ 709  imes 10$	$0.754\ 045\ 685  imes 10$	$0.161\ 227\ 439 \times 10^2$	$\mathbf{J} \ \mathbf{kg}^{-1} \ \mathbf{K}^{-1}$
$(\partial^2 g/\partial T^2)_{p,S}$	$-0.142\ 987\ 096\times 10^2$	0.488 076 974	$-0.138\ 106\ 326\times 10^2$	$\mathrm{J}~\mathrm{kg}^{-1}~\mathrm{K}^{-2}$
$(\partial^2 g/\partial p \ \partial T)_S$	$0.202\ 974\ 451  imes 10^{-6}$	$0.466\ 284\ 412\times 10^{-7}$	$0.249\ 602\ 892  imes 10^{-6}$	$m^3 kg^{-1} K^{-1}$
$(\partial g/\partial S)_{p,T}$	0	$-0.545~861~581\times 10^4$	$-0.545~861~581\times 10^4$	${ m J~kg}^{-1}$
$(\partial^2 g/\partial p \ \partial S)_T$	0	$-0.640\ 757\ 619\times 10^{-3}$	$-0.640\ 757\ 619\times 10^{-3}$	$m^3 kg^{-1}$
v	$0.956\ 686\ 939\times 10^{-3}$	$-0.229\ 123\ 842\times 10^{-4}$	$0.933\ 774\ 555  imes 10^{-3}$	$m^3 kg^{-1}$
и	-0.282 725 255	$-0.236\ 936\ 788\times 10^1$	$-0.265\ 209\ 313 \times 10^{1}$	kJ k $g^{-1}$
h	$0.953\ 859\ 687  imes 10^2$	$-0.466\ 060\ 630\times 10^{1}$	$0.907\ 253\ 624 \times 10^2$	kJ k $g^{-1}$
S	$-0.858\ 228\ 709\times 10^{-2}$	$-0.754\ 045\ 685\times 10^{-2}$	$-0.161\ 227\ 439\times 10^{-1}$	kJ $\mathrm{kg}^{-1}$ $\mathrm{K}^{-1}$
$c_p$	$0.390\ 569\ 252  imes 10^1$	-0.133 318 225	$0.377\ 237\ 430 \times 10^{1}$	kJ $\mathrm{kg}^{-1}$ $\mathrm{K}^{-1}$
W	$0.157\;552\;544 \times 10^4$	_ a	$0.162\ 218\ 081  imes 10^4$	${ m m~s}^{-1}$
$\mu_{ m W}$	$0.977\ 302\ 204 \times 10^2$	$-0.240\ 897\ 806 \times 10^{1}$	$0.953\ 212\ 423 \times 10^2$	kJ kg <sup>-1</sup>

<sup>a</sup> This value cannot be computed from  $g^{S}$  alone because it is a nonlinear expression in g

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**Table A2** Boiling temperatures  $T_b$  in K of seawater for selected points (p,S) calculated by iteration from Eq. (6) or from Eq. (7)

Pressure <i>p</i>			Sal	inity S in kg kg	g <sup>-1</sup>		
in MPa	0	0.02	0.04	0.06	0.08	0.10	0.12
0.001	280.12	280.27	280.43	280.61	280.80	281.01	281.25
0.005	306.03	306.21	306.41	306.63	306.87	307.14	307.44
0.010	318.96	319.16	319.38	319.62	319.89	320.19	320.52
0.015	327.12	327.34	327.57	327.83	328.11	328.42	328.78
0.020	333.21	333.44	333.68	333.95	334.24	334.57	334.94
0.025	338.11	338.35	338.60	338.88	339.18	339.52	339.91
0.030	342.25	342.49	342.75	343.03	343.34	343.69	344.09
0.035	345.83	346.08	346.34	346.63	346.95	347.31	347.72
0.040	349.01	349.26	349.53	349.83	350.15	350.52	350.94

**Table A3** Freezing temperatures  $T_f$  in K of seawater for selected points (*p*,*S*) calculated by iteration from Eq. (11) or from Eq. (12)

Pressure <i>p</i>	Salinity S in kg kg <sup>-1</sup>						
in MPa	0	0.02	0.04	0.06	0.08	0.10	0.12
0.001	273.1600	272.0823	270.9611	269.7618	268.4609	267.0397	265.4900
0.005	273.1597	272.0820	270.9608	269.7615	268.4606	267.0394	265.4897
0.01	273.1593	272.0816	270.9604	269.7612	268.4602	267.0390	265.4893
0.05	273.156	272.079	270.957	269.758	268.457	267.036	265.486
0.1	273.153	272.075	270.954	269.754	268.453	267.032	265.482
1	273.09	272.01	270.89	- <sup>a</sup>	- <sup>a</sup>	- <sup>a</sup>	- <sup>a</sup>
10	272.40	271.32	270.20	- <sup>a</sup>	- <sup>a</sup>	- <sup>a</sup>	- <sup>a</sup>
20	271.61	270.53	269.40	- <sup>a</sup>	- <sup>a</sup>	- <sup>a</sup>	- <sup>a</sup>
50	269.06	267.97	266.82	- <sup>a</sup>	- <sup>a</sup>	- <sup>a</sup>	- <sup>a</sup>
100	264.21	263.09	261.92	_ <sup>a</sup>	_ a	- <sup>a</sup>	- <sup>a</sup>

<sup>a</sup> This state point is out of range of validity of Eq. (1).

**Table A4** Triple-point temperatures  $T_t$  and related triple-point pressures  $p_t$  of seawater for selected salinities *S* calculated by iteration from both Eqs. (6) and (11) or from both Eqs. (7) and (12)

Salinity S in kg kg <sup><math>-1</math></sup>	0	0.02	0.04	0.06	0.08	0.10	0.12
Triple-point temperature $T_{\rm t}$ in K	273.16	272.08	270.96	269.76	268.46	267.04	265.49
Triple-point pressure $p_t$ in kPa	0.61168	0.55953	0.50961	0.46073	0.41257	0.36524	0.31932