

# New Correlation for the Viscosity of Normal Butane

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  - Problems with consistency
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  - Viscosity in the fluid region
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# Motivation – Problems and Tasks

## Convenience for engineers

- Use of a Standard Database Program for Thermophysical Properties – consistent with respect to thermodynamic and transport properties: REFPROP<sup>1</sup>
- Consistency of the formulations for water:
  - Equation of state (EOS): Wagner and Pruß (2002)<sup>2</sup>
  - Viscosity  $\eta$ : Huber *et al.* (2009)<sup>3</sup>
  - Thermal conductivity  $\lambda$ : Huber *et al.* (2012)<sup>4</sup>

<sup>1</sup> Lemmon, E. W., Huber, M. L., and McLinden, M. O., Standard Reference Data Program, National Institute of Standards and Technology, Gaithersburg (2013).

<sup>2</sup> Wagner, W. and Pruß, A., *J. Phys. Chem. Ref. Data* **31**, 387-535 (2002).

<sup>3</sup> Huber, M. L., Perkins, R. A., Laesecke, A., Friend, D. G., Sengers, J. V., Assael, M. J., Metaxa, I. M., Vogel, E., Mares, R. and Miyagawa, K., *J. Phys. Chem. Ref. Data* **38**, 101-125 (2009).

<sup>4</sup> Huber, M. L., Perkins, R. A., Friend, D. G., Sengers, J. V., Assael, M. J., Metaxa, I. M., Miyagawa, K., Hellmann, R. and Vogel, E., *J. Phys. Chem. Ref. Data* **41**, 1-23 (2012).

## Ethane: EOS, $\lambda$ – inconsistent

- Correlations recommended in REFPROP
  - EOS Bücker and Wagner (2006)<sup>5</sup>
  - $\eta, \lambda$  Friend *et al.* (1991)<sup>6</sup>
- Consistent for  $\eta$ : new viscosity-surface correlation published in *J. Phys. Chem. Ref. Data*<sup>7</sup>

## Propane: EOS, $\lambda$ – inconsistent

- Correlations recommended in REFPROP
  - EOS Lemmon *et al.* (2009)<sup>8</sup>
  - $\eta$  Vogel *et al.* (1998)<sup>9</sup>
  - $\lambda$  Marsh *et al.* (2002)<sup>10</sup>
- Consistent for  $\eta$ : manuscript submitted to *J. Phys. Chem. Ref. Data*

<sup>5</sup> Bücker, D. und Wagner, W., *J. Phys. Chem. Ref. Data* **35**, 205-266 (2006).

<sup>6</sup> Friend, D. G., Ingham, H., und Ely, J. F., *J. Phys. Chem. Ref. Data* **20**, 275-347 (1991).

<sup>7</sup> Vogel, E., Span, R., and Herrmann, S., *J. Phys. Chem. Ref. Data* **44**, 043101 (2015).

<sup>8</sup> Lemmon, E. W., McLinden, M. O., and Wagner, W., *J. Chem. Eng. Data*, **54**, 3141-3180 (2009).

<sup>9</sup> Vogel, E., Küchenmeister, C., Bich, E., and Laesecke, A., *J. Phys. Chem. Ref. Data*, **27**, 947-970 (1998).

<sup>10</sup> Marsh, K., Perkins, R., and Ramires, M. L. V., *J. Chem. Eng. Data*, **47**, 932-940 (2002).

# Problems with consistency

## Normal Butane: EOS, $\eta$ , $\lambda$ – inconsistent

- Correlations recommended in REFPROP

EOS Bücker and Wagner (2006)<sup>11</sup>

$\eta$  Vogel *et al.* (1999)<sup>12</sup>

$\lambda$  Perkins *et al.* (2002)<sup>13</sup>

- Characterization

EOS classical including the critical region, an additional parametric crossover EOS not needed

$\eta$  not including a critical enhancement, but using an old-fashioned classical MBWR

$\lambda$  including a critical enhancement according to a simplified crossover model by Olchoway and Sengers (1988)<sup>14</sup>, but again based on an old-fashioned classical MBWR

<sup>11</sup> Bücker, D. and Wagner, W., *J. Phys. Chem. Ref. Data* **35**, 929-1019 (2006).

<sup>12</sup> Vogel, E., Küchenmeister, C., Bich, E., *High Temp. - High Pressures*, **31**, 173-186 (1999).

<sup>13</sup> Perkins, R., Ramires, M. L. V., Nieto de Castro, C. A., and Cusco, L. *J. Chem. Eng. Data*, **47**, 1263-1271 (2002).

<sup>14</sup> Olchoway, G. A. and Sengers, J. V., *Phys. Rev. Lett.*, **61**, 15-18 (1988).

# Correlation method using structure optimization

## Selection criteria

- Combination of different terms
- Requirement of reliable experimental data
- Use of simple functional dependencies, e.g.,  $\eta = \eta(T, \rho)$
- Viscosity-surface correlations for normal butane:
  - Vogel *et al.* (1999) → recommended within REFPROP
  - Quiñones-Cisneros and Deiters (2006)<sup>15</sup> → not recommended

## Procedure

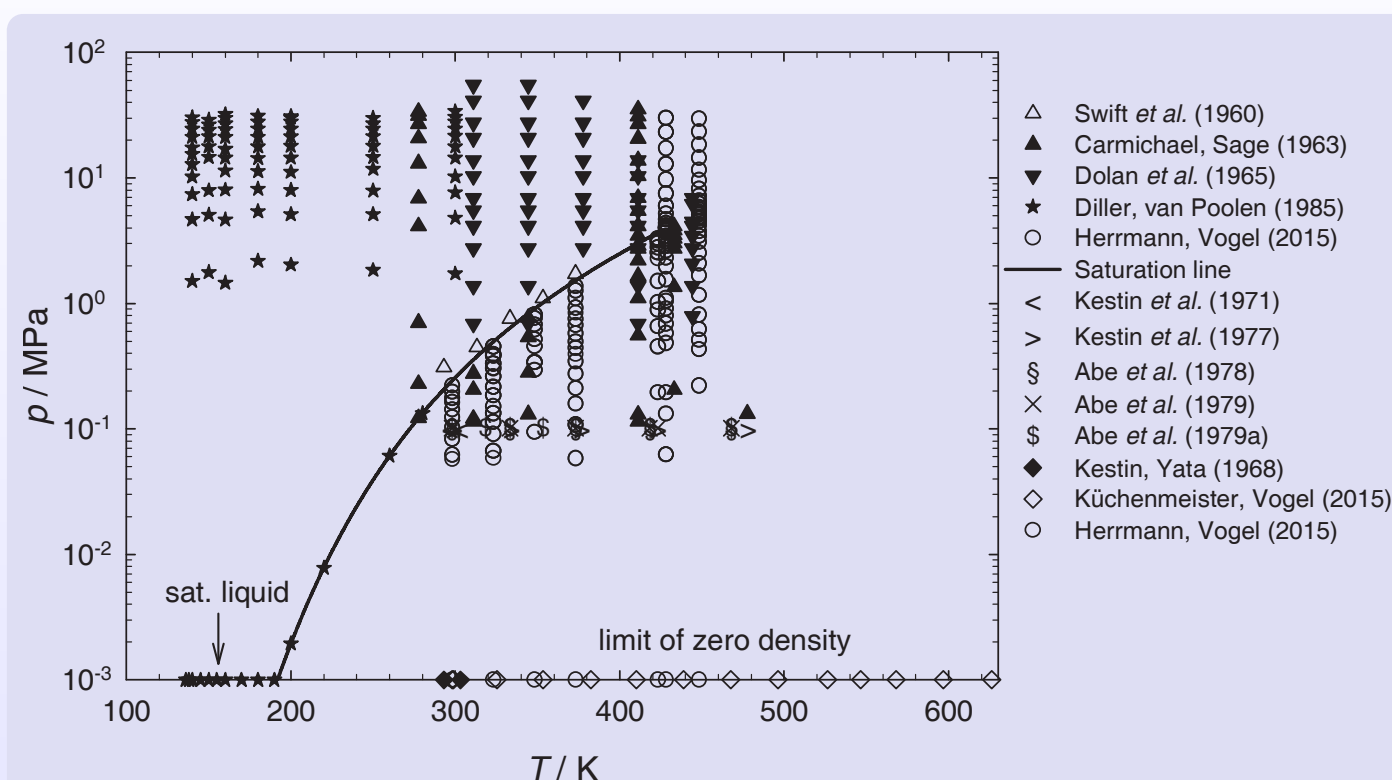
- Evaluation and classification of all available viscosity data
- Selection of terms for the complete fluid range of thermodynamic states including the near-critical region
- Assessment of the resulting correlation using statistical parameters and adequate description of experimental data

<sup>15</sup> Quiñones-Cisneros, S. E. and Deiters, U. K., *J. Phys. Chem. B* **110**, 12820-12834 (2006).

## Normal Butane – Primary Experimental Viscosity Data

Authors	Year	Method <sup>16</sup>	Number of points	$T$ K	$\rho$ kg m <sup>-3</sup>	$\Delta\eta/\eta$ %
Kestin, Yata	1968	OD	2	293–303	0	0.4
Küchenmeister, Vogel	2015 <sup>17</sup>	OD	14	298–626	0	0.3
Herrmann, Vogel	2015	VW	7	298–448	0	0.3
Kestin <i>et al.</i>	1971	OD	2	297–303	2–3	0.4
Kestin <i>et al.</i>	1977	OD	5	299–478	1–2	0.4–1.0
Abe <i>et al.</i>	1978	OD	5	298–468	2	0.4–1.0
Abe <i>et al.</i>	1979	OD	6	298–468	2	0.4–1.0
Abe <i>et al.</i>	1979	OD	7	298–468	2	0.4–1.0
Swift <i>et al.</i>	1960	FC	5	293–373	468–579	2.5
Dolan <i>et al.</i>	1963	C	50	311–444	13–623	2.5
Carmichael, Sage	1963	RC	45	278–478	2–631	2.5
Diller, van Poolen	1985	OQC	89	136–300	573–742	2.5
Herrmann, Vogel	2015	VW	289	298–448	1–498	0.5

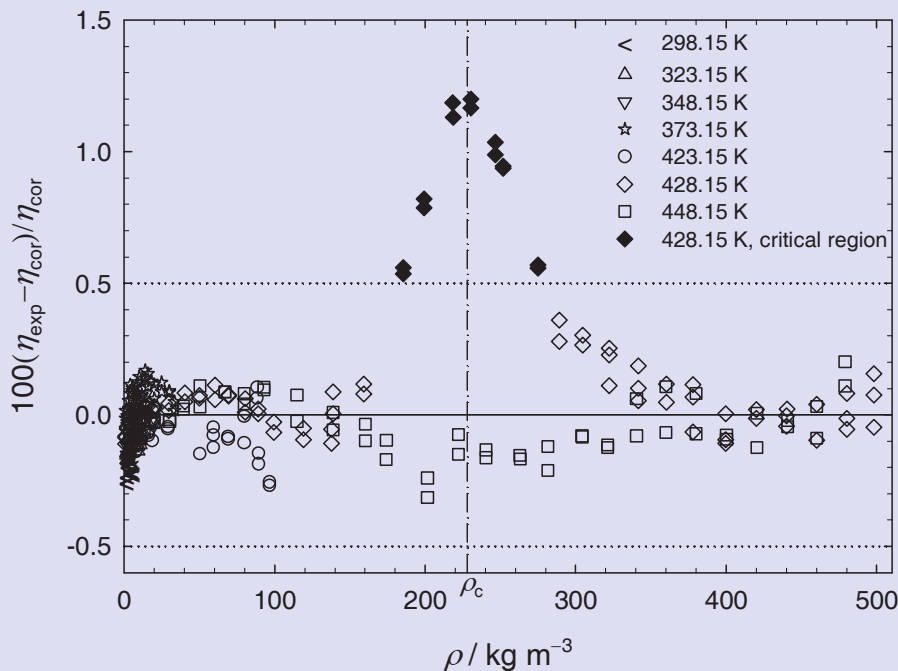
<sup>16</sup>C, capillary; FC, falling cylinder; OD, oscillating disk; OQC, oscillating quartz crystal; RC, rotating cylinder; VW, vibrating wire  
<sup>17</sup> re-evaluated data

Normal Butane –  $p, T$  diagram with primary experimental data

# Normal Butane – Correlation without critical enhancement

## New data for normal butane of Herrmann and Vogel (2015)<sup>18</sup>

- Deviations up to +1.20 % near critical density ( $\rho_c = 228.0 \text{ kg m}^{-3}$ )



<sup>18</sup> Herrmann, S. and Vogel, E., *J. Chem. Eng. Data*, **60**, 3703-3720 (2015).

## Critical enhancement according to Bhattacharjee *et al.* (1981)<sup>19</sup>

- Viscosity  $\eta$  corresponds to an asymptotic power-law divergence:

$$\eta \approx \eta_b (Q_0 \xi)^{z_\eta} .$$

- Critical enhancement represents a multiplicative anomaly:

$$\eta_c = \eta_b [(Q_0 \xi)^{z_\eta} - 1] .$$

- Crossover is needed  $\rightarrow$  complete global solution by Olchowy and Sengers (1988) for the mode-coupling theory:

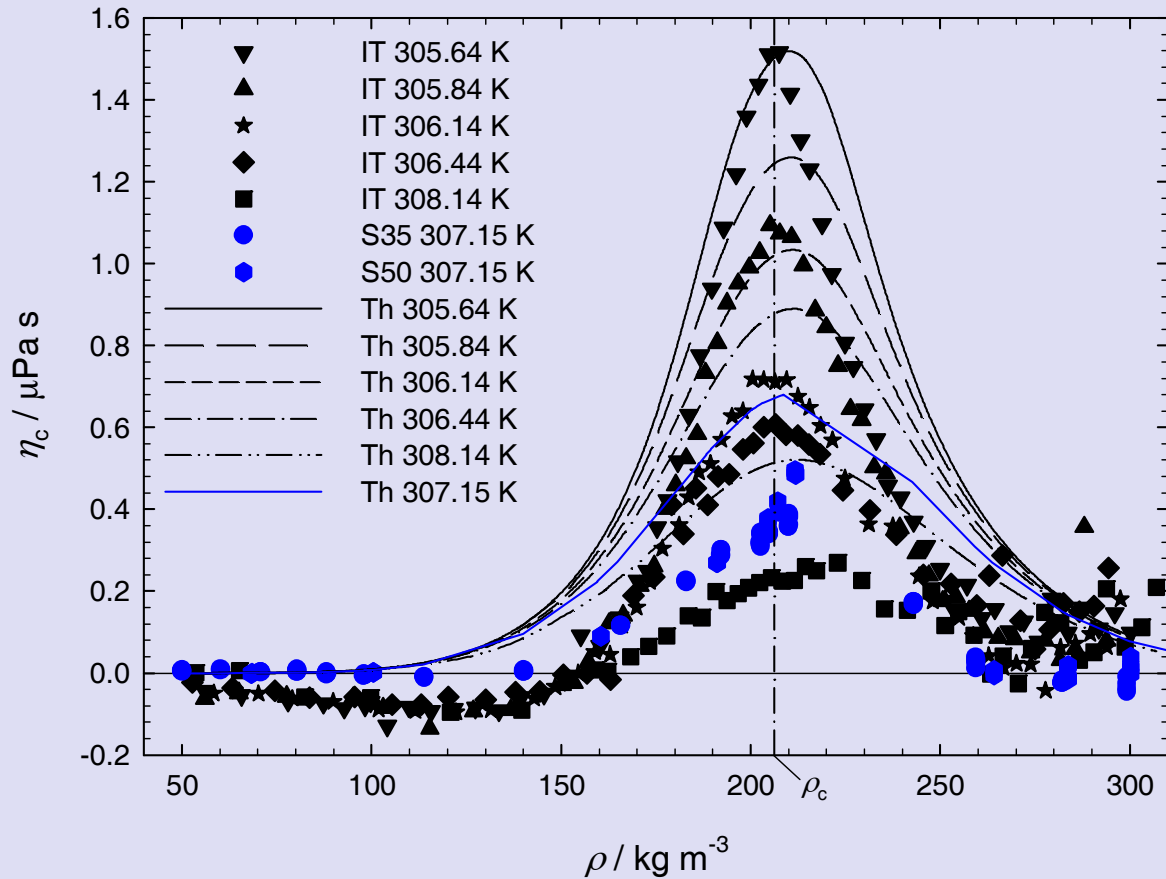
$$\eta_c = \eta_b [\exp(z_\eta H) - 1] .$$

- Simplified closed-form solution earlier developed (Bhattacharjee *et al.*)  $\rightarrow$  recently used for IAPWS water (Huber *et al.*, 2009):

$$\eta_c = \eta_b [\exp(z_\eta Y) - 1] .$$

<sup>19</sup> Bhattacharjee, J. K., Ferrell, R. A., Basu, R. S., and Sengers, J. V., *Phys. Rev. A* **24**, 1469-1475 (1981).

## Critical enhancement – ethane: theory vs. experiment



## Viscosity-surface correlation for normal butane

- Reduced quantities:  $\tau = \frac{T_c}{T}$ ,  $\delta = \frac{\rho}{\rho_c}$
- Bank of terms for separate zero-density viscosity correlation:

$$\frac{\eta_{0,\text{bank}}(T)}{\mu\text{Pa s}} = \sum_{i=0}^{-8} A_{0,i} \tau^i \quad \text{Result: } A_{0,0}, A_{0,-1}, A_{0,-6}, A_{0,-7}.$$

- Bank of terms for the total correlation:

$$\begin{aligned} \frac{\eta_{\text{bank}}(T, \rho)}{\mu\text{Pa s}} = & \frac{\eta_0(T)}{\mu\text{Pa s}} A_0 + \sum_{i=0}^8 \sum_{j=1}^{20} A_{ij} \tau^i \delta^j + \sum_{k=0}^5 \sum_{l=1}^5 A_{kl} \tau^k \delta^l e^{-\delta} \\ & + \sum_{m=0}^1 A_m \tau \delta \mu_m e^{-\beta_m (\delta - \gamma_m)^2 - \varepsilon_m |\tau - \zeta_m|}. \end{aligned}$$

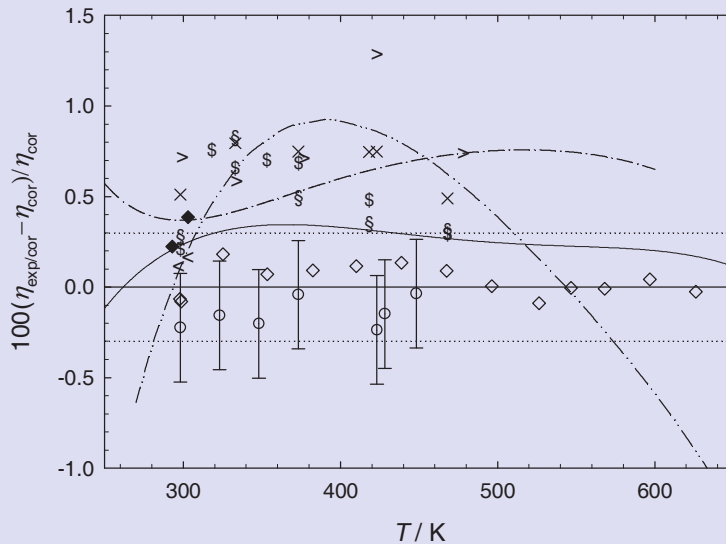
- Final result for normal butane:

$$\begin{aligned} \frac{\eta_{\text{cor}, n\text{-C}_4\text{H}_{10}}(T, \rho)}{\mu\text{Pa s}} = & \sum_{i=1}^{10} A_i \tau^{t_i} \delta^{d_i} + \sum_{i=11}^{13} A_i \tau^{t_i} \delta^{d_i} e^{-\delta} \\ & + \sum_{i=14}^{15} A_i \tau \delta e^{-\beta_i (\delta - 1)^2 - \varepsilon_i |\tau - 1|}. \end{aligned}$$

# Comparison equation vs. experiment

## Viscosity in the limit of zero density and at low densities

- Agreement within the experimental uncertainty
- Error bars:  $\pm 0.3\%$



◆, ◇, ○ experimental data in the limit of zero density

<, >, §, x, \$ experimental data at atmospheric pressure

- · - Younglove, Ely (1987)<sup>20</sup>

— Vogel *et al.* (1999)

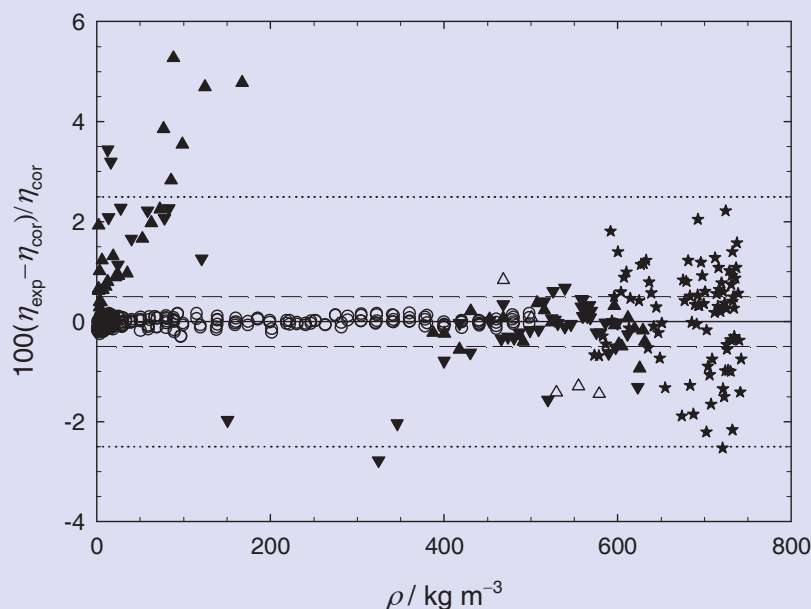
- - - Quiñones-Cisneros, Deiters (2006)

<sup>20</sup> Younglove, B. A. and Ely, J. F., *J. Phys. Chem. Ref. Data* **16**, 577-798 (1987).

# Comparison equation vs. experiment

## Viscosity in the fluid region

- New data dominant
- Large deviations particularly at small densities for earlier primary data
- Deviations of data in the near critical region  $< \pm 0.15\%$



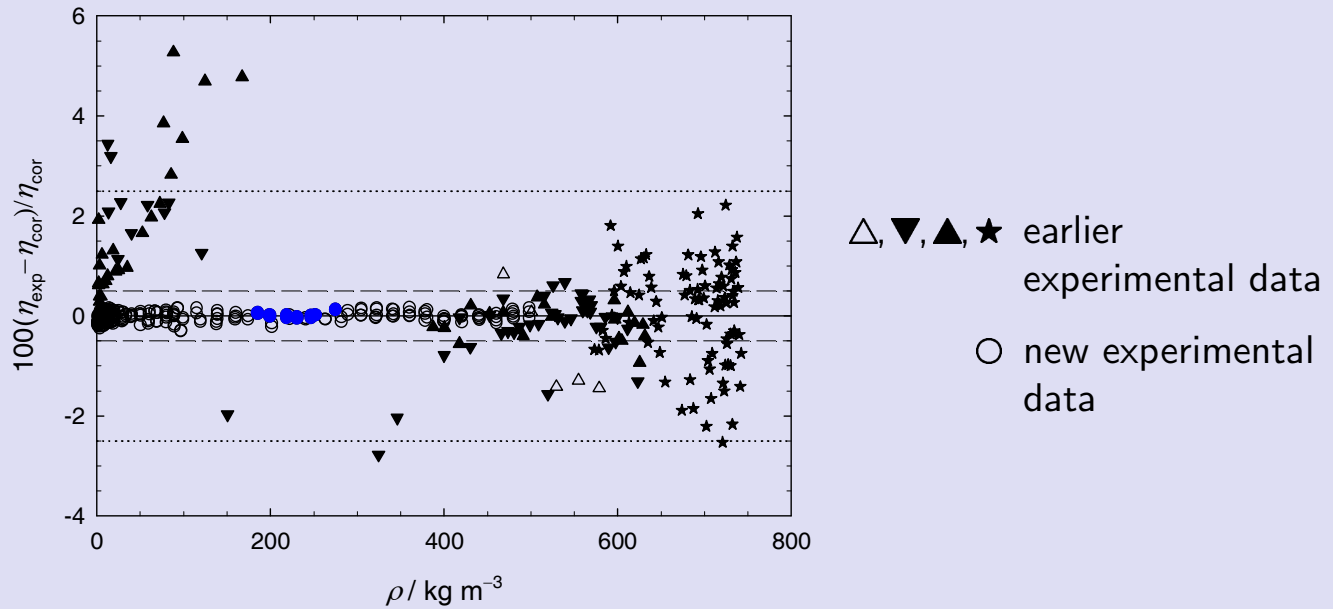
△, ▽, ▲, ★ earlier experimental data

○ new experimental data

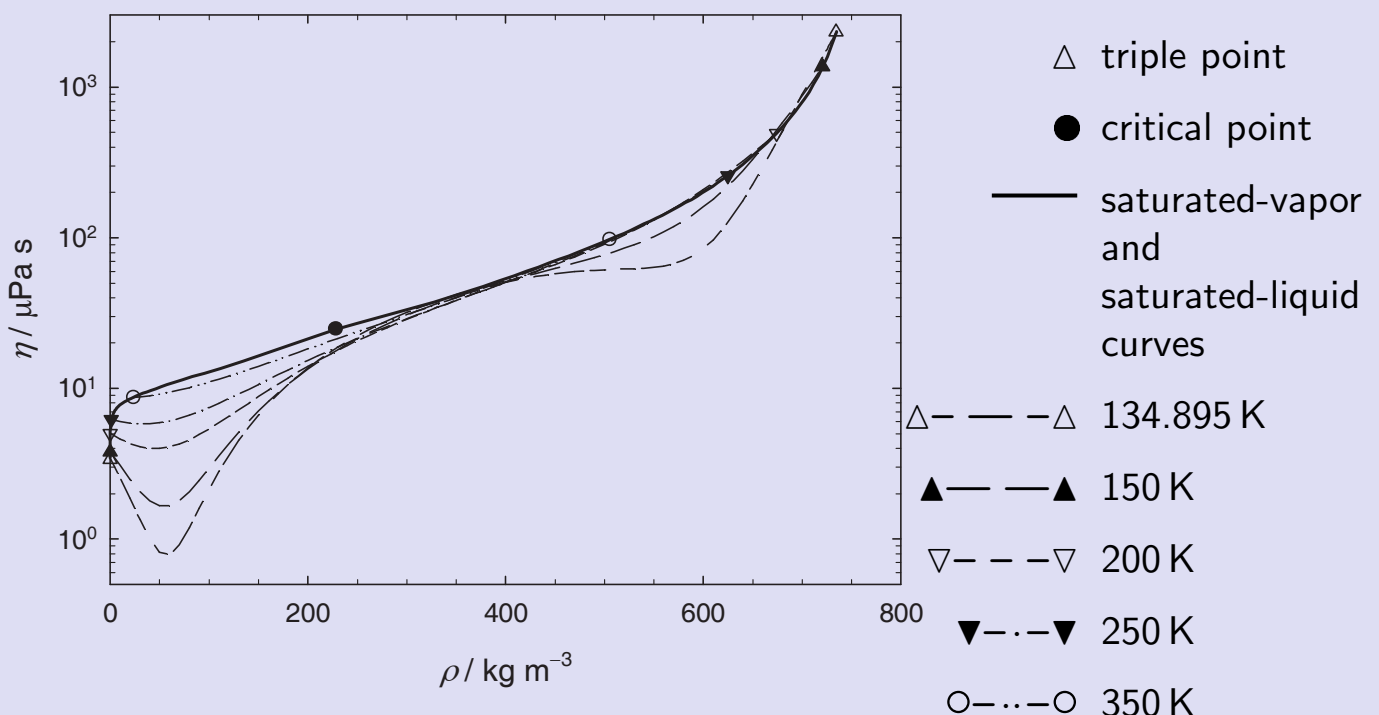
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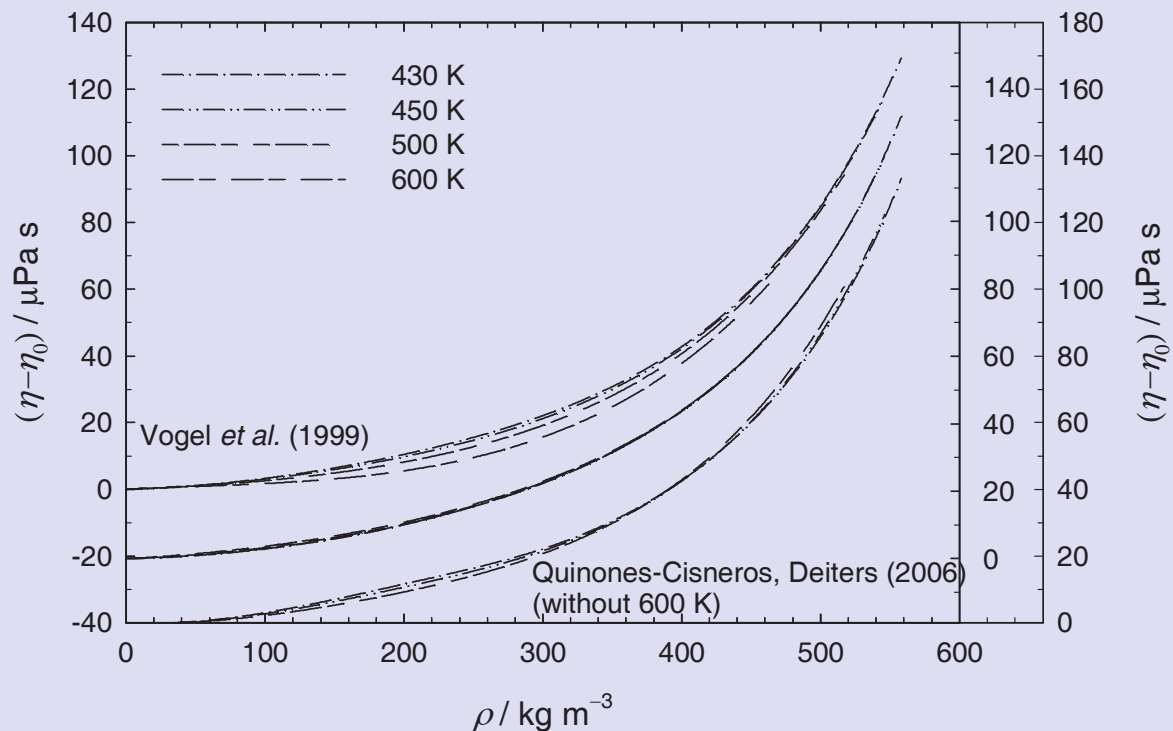
# Correlation in the two-phase region





# Consistency test using behavior of $\eta_{Res}$

Comparison to viscosity-surface correlations from literature:  
Vogel *et al.* (1999) and Quiñones-Cisneros and Deiters (2006)



## Conclusion and Outlook

# Conclusion and Outlook

- New viscosity-surface correlation was generated for normal butane based on new precise experimental viscosity data
- The structure-optimization method of Setzmann and Wagner (Ruhr-Universität Bochum) was used
- The viscosity was correlated as  $\eta(T, \rho)$
- Critical enhancement was included using new data of Herrmann and Vogel

Theory: divergence at the critical point

Correlation: finite values when approaching the critical point due to used experimental data from the near-critical region

- Further work on isobutane

→ precise data using a vibrating-wire viscometer are available

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